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Mathematical and Geometric proportions Used in Tiles of the Isfahan Mosques in Safavid Era with Respect to Optimization of Construction Materials

Nima Valibeig*
Shafagh Tavakkoli**
Nafise Khodadadi***

Abstract
Geometrical and mathematical proportions play a fundamental role in art and architecture. Persian architecture has been always closely related to geometry. This is easily visible in decorations and patterns of surfaces. Persian architects made utmost effort to apply construction materials as economically as possible. Studying the tile works in terms of form and geometrical characteristics reveals how tiles have been applied in mosques. In this respect, Esfahan is one of the cities of Iran with an amazing variety in terms of tile works used in its mosques. The spiritual beauty and geometrically skilful representations reach its zenith in Esfahan's mosques especially at the time it was the capital of an Empire- Safavid era. Applying tiles and materials economically has had a direct relationship with geometric mastery. Most studies in this connection have dealt with symbolic representations and drawing techniques of Persian tile works. This study, however, aimed to examine geometric characteristics of tiles, the quantity of them and the number of workforce needed to apply tiles on the wall surfaces of Isfahan mosques of Safavid era. Through doing library research and geometric analyses, the number of workforce and tiles used was estimated. Then, several tile fretworks were dismantled into its constituents. The area, perimeter and occurrence frequency of each form were examined through applying special soft wares. In addition, the work force needed to make each form was estimated and compared among the different fretworks. The number of tiles applied in terms of their color was determined in each tile inscription. In order to draw the patterns, the techniques used by the distinguished experts (Lorzade, Maher-o-Naghs and Shaarbaf) were followed. In fact, the present study tried to reveal Persian architects' mastery over geometry. Since the majority of similar studies have been done on famous mosques, it was decided that less famous mosques be examined to, at the same time, make other researchers encourage study less historically distinguished buildings as well. In this connection, two mosques- Khayyatha Mosque and Zolfaghar-e-Neemavard Mosque, both built in Safavid era were studies. The two mosques have various brick and tile patterns. Tile fretworks, composed of formal elements, have their own geometrical characteristics. By studying the constituents of the fretworks, the geometrical and mathematical proportions were obtained. The effect of size and form of tile work on workforce and used materials was recognized. In addition, the most commonly used tile work patterns in Khayyatha mosque and Zolfaghar-e-Neemavard mosque was found. This study may help Persian traditional tile fretworks and tile decorations perpetuate.

Keywords
Geometric proportions, mathematical proportions, workforce, Khayyatha mosque, Zolfaghar-e-Neemavard mosque.

*. Ph. D. in Restoration of Historical Buildings. Assistant Professor, Isfahan Art University. n.valibeig@aui.ac.ir
**. M. A. in Restoration of Historical Buildings. Isfahan Art University. tavakoli.shafagh@gmail.com
***. M. A. in in Restoration of Historical Buildings. Isfahan Art University. khodadadi.nafise@gmail.com

Introduction

Geometry can be basically considered as the corner stone of Creation (Ayatollahi, 2004: 228). The foundation of Persian traditional architecture is based on geometry and thorough drawing techniques The importance of this has been so much so that it was highly imperative for the architects to master mathematical rules as well as various drawing techniques in practice (Molavi & Ghasemzadeh, 2002:12). Generally speaking, the essence of art and architecture is geometry and proportions (Bemanian, 2011:19). Geometric patterns constitute one of the salient features of Islamic art and architecture (www.broug.com). What is known as geometry may be divided into two areas: theoretical and applied. The former deals with lines, surfaces and volumes and the latter addresses the application of geometric proportions and patterns onto wood, metal, brick, etc. (Farabi, 2002: 77). Mainly based on decorations, Persian art represents shapes and figures each of which symbolizes entities of the external world (Pope, 2001:1). It is an innovative expression of geometric shapes and patterns (Johnson, 2009:275) a major part of which includes the skilful application of geometric patterns onto tile and brick surfaces (Rogers, 2008: 8). Of the principles of Persian architecture is avoidance of vanity represented intelligently onto the surface decorations. In tile works, for example, the waste of materials has been always close to minimum. The history of both Khayyatha Mosque and Neemavard Mosque, placed in Isfahan Bazaar, dates back to Safavid era.

The mosques enjoy a wide variety of decorative tile and brick patterns along with different beautiful tile fretworks. Consisted of diverse formal and geometric modules (called as mohre), each piece of fretwork has its own geometric specifications.

Through examination of individual tile modules in fretworks, the geometric proportions applied may be identified. Similarly, by analyzing the tile patterns, the geometric specifications applied in tile fretworks and the number of tile modules used in mosque may be determined. This may paves the way to preserve the existing tile works and to revive such Persian tile works in today's construction activities.

The present article tried to answer the following questions:

What kinds of tile fretworks have been more commonly used in Khayyatha Mosque and Neemavard Mosque?

How is it possible, based on mathematical calculations, to identify the workforce involved in making tile fretworks of the mosque?

Review of Literature

One of the oldest works on geometry has been written by Abū al-Wafā, Muḥammad ibn Muḥammad ibn Būzjānī, Persian mathematician and astronomer, living in the 10th century AD. Būzjānī's work has illustrated how to draw a wide variety of geometric shapes (Būzjānī, 2010). Also, in his great work a treatise on vault and Azaj, Ghiyathoddin Jamshid Kashani has explained different ways to measure dimensions and areas of various geometric shapes (Kashani, 1987). So far, tile fretworks have been studied extensively from different aspects. Some masters, for example, have studied tile fretworks in terms of drawn patterns (Lorzade, Maher-o-Naghsh, Shaarbaf, Shafaie, Fereshtenejad…); Some in terms of construction methods as well as implementation techniques (Peernia and Bozorgmehri) and still others in terms of different ways of constructing wooden fretworks (Tasooji). Also, the authors of the present article carried out a similar research on Masjed-e- Safavi-ye-Shishe-ye-Isfahan (Glass Safavid Mosque of Isfahan) the objective of which was to identify, compare and develop mathematical proportions hidden in the patterns. This study, however, aimed to identify tile fretworks and their mathematical as well as geometric proportions in Khayyatha Mosque and Neemavard Mosque.

Methodology

The methodology has been based on library, analytic and field studies. The two mentioned mosques were selected, among many others, because they enjoyed a wide variety of decorative tile works and brick works. The statistical population included all kinds of fretworks used in the two mosques. The picture of each pattern was taken. Also, patterns were dismantled mathematically and drawn through computerized modeling. Then, individual fretworks were analyzed into its constituents (mohre). The surface, perimeter and the frequency percentage of individual forms were calculated using relevant soft wares. Then, the workforce needed for different tile fretworks for each pattern was estimated and compared with other patterns. Also, the number of tiles used for each given color in various inscriptions was determined. In this article, the drawing methods used by distinguished masters (Lorzade, Maher-o-Naghsh and Shaarbaf) were followed.

Gere (knot)

Gere (knot/ fretwork) refers to interwoven patterns existing for a very long time. (Fereshtenejad, 2009:297).

The geometric patterns of the fretworks have been used since centuries ago onto the walls, ceilings, openings, domes and minarets (Embi & Abdullahi, 2012: 2). In addition, fretworks include various textured surfaces of regular geometric shapes arranged in a well-balanced fashion (Navaie & Hajighasemi, 2011:176). Fretworks are mainly based on linear, circular and square forms organized by repetition and combination (The Metropolitan Museum of Art, 2004:10). The important point concerning the fretworks is that each piece of fretwork is restricted to its own frame in that the overall form of the fretwork is not independent of its frame/background (Shaarbaf, 2006:9). Fretworks are of two and three dimensional kinds the constituents of which are similar (Najib Oghlou, 2010:31). In some techniques, chains of gere (given number of knots) are arranged sequentially in an alternate fashion, one knot recessed and the other one projected. This alternate fashion may be also repeated in terms of color. In tile work, often a blue or turquoise stripe, called as daval, separates the other tile stripes from each other (Lorzade, 2010:142).

Nowadays, those who intend to revive traditional tile works in buildings apply mainly regular polygonal networks named as repetitive patterns and raised tile work (Webster, 2013: 88). Such patterns are basically composed of geometric shapes. The tile makers were quite familiar with applied geometric concepts (Kaplan, 2004: 98).

The Mosques of Isfahan in Safavid Era

Khayyatha Mosque

Khayyatha Mosque, also locally called as Masjed-e-pa derakht-e-Soukht-e- Zanjani (meaning literally as Zanjani's Burnt Log Mosque) may be considered as one of the small-scale mosques of Safavid era in Isfahan. The only recognizable date of the mosque is the one inscribed on its old door reading as 975 Hijri / 1567 (Hajighasemi, 2004: 423).

The mosque was built by order of King Tahmasb's son, Abol Fath Sultan Ahmad Mirza, whose name is visible in the inscription over the mosque door (Rafiei Mehrabadi, 1973: 340). In August 20th, 1998, Khayyatha Mosque was recorded, under registration number 2154, as one of the National Monuments of the country (Report on Iranshahr Architectural History).

Zolfaghār-e-Nimavard Mosque

Located in Nimavard Neighborhood, Isfahan, the mosque dates back to Safavid era. According to the inscription over the mosque door written in Sulus calligraphy, the mosque was built by Sheikh Mohammad Safi in the reign of Shah Tahmasb I, in 950 Hijri/ 1543 (Honarfar, 1971:384). The inscription is made of white mosaic on a turquoise background.

The Characteristics of the Fretwork Patterns Applied in the Two Mosques

Various fretwork patterns have been applied in the two mentioned mosques. Whereas some of the patterns including kond-e-sorme dan ghenas koochak ; Gol sabounaki; shamse ; squared shahar soli as well as shesh tond-e-zamine hasht are totally made of tiles, others including sekron and bazoobandi are made of tile-and-brick combinations. One of the most beautiful geometric patterns commonly used in the fretworks is the star-like (solar) form (Lee, 1995:18).

The fretwork of kond-e- dow panj

The fretwork of kond-e- dow panj is the basic form of many other fretworks especially different kinds of fretwork dah (ten) (Lorzadeh 2010: 145). This pattern has been often over accomplished onto the main portals of the mosques (Fig. 1).

In table 1, the perimeter, area and frequency of each form of the fretwork of kond-e- dow panj have been given separately. The workforce is the result of perimeter multiplied by frequency (number); the

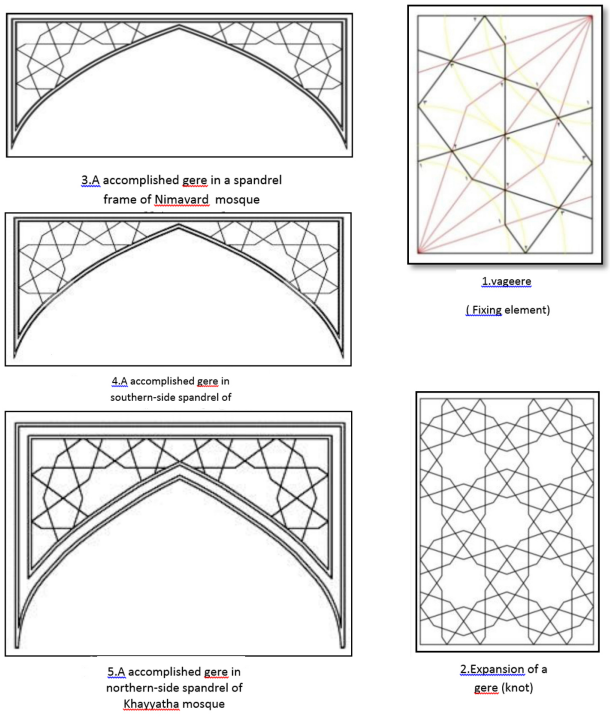


Fig. 1. Fretwork kond-e- dow panj. Source: authors.

tile pieces are not in prefabricated form, but rather, each tile piece has to be cut by the master so that the desired size and form is obtained. The workforce for the fretwork of kond-e- dow panj has been calculated in different accomplished frames (Table 1).

The fretwork of shesh tond-e-zamine dah

This role is made up of five vertebrae (Fig. 2-a). Given that the mentioned fretwork has been accomplished in three different background frames in Khayyatha mosque, and in one single background frame in Nimavard mosque, the work force for individual patterns used in the fretwork of shesh tond-e-zamine dah has been calculated (Table 2).

The fretwork of shesh tond-e-zamine hasht

This fretwork, totally in tile, has been accomplished in the western side of the Khayyatha mosque. In its regular octagonal pattern, the shape of Ali (the name of the first Imam of Moslems) is visible (Fig. 2-b). The area, perimeter and frequency of individual shapes used in this fretwork have been calculated and then the workforce needed was estimated. Given that the mentioned fretwork has been accomplished in tile, the number of tiles used for each distinct color was determined separately which was equal to the total frequency of mohres of a given color multiplied by the area (Table 2).

The fretwork of Shamse-va-Chahar Soli-e-Moraba-Dar.

This pattern has been accomplished in Nimavard mosque (Fig. 3).The fretwork of Shamse-va-Chahar Soli-e-Moraba-Dar has been accomplished in two frames in Nimavard mosque. In one of the frames, the modules have been accomplished in rectangular forms. Also, in order to calculate the amount of workforce and the total area of tile pieces used, the

Table 1.The geometric specifications applied in the fretwork of kond-e-dow panj. Source: authors.

perimeter Frequency	Frequency * Area (cm^2)	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre	
485	5897	2.5	194	2359		Neemavard Mosque
760	4180	9.5	80	440		
1248	3552	24	52	148		
104	669	1	104	669		
4173=2150+1469+1254 = 3-2 workforce in Figure			Workforce= frequency * Area			

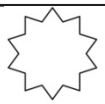
perimeter Frequency *	Frequency * Area(cm^2)	Frequency in the spandrel	Perimeter (cm)	Area (cm^2)	Mohre	
1097	30162	2.5	439	12065		Khayyatha Mosque
612.5	9355	2.5	245	3742		
1672	20330	9.5	176	2140		
931	6308	9.5	98	664		
2712	81700	24	113	723		
1536	5520	24	64	230		
228	3253	1	228	3253		
128	1022	1	128	1022		
5709=228+2712+1672+1097= 4-2 workforce in Figure 3207=128+1536+931+612.5=5-2 workforce in Figure			Workforce= frequency * Area			

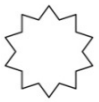
fretwork was dismantled into its modules and the area and surface of the modules were calculated (Table 2).

The fretwork of kond-e-sorme dan ghenas koochak

This fretwork has been accomplished in the spandrels

Table 2.The geometric specifications applied in the fretworks of shesh tond-e-zamine dah and shesh tond-e-zamine hasht. Source: authors.

Frequency × perimeter	Frequency * Area (cm^2)	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre		
400	2460	2.5	160	984		The fretwork of *shesh tond-e-zamine dah*	Neemavard Mosque
1032	2848	8	129	356			
2232	13824	24	93	576			
558	1542	6	93	257			
1160	200	20	58	160			
2-A-3 workforce in Figure 5382=1160+558+2232+1032+400=			Workforce= frequency * Area				

Frequency × perimeter	Frequency * Area (cm^2)	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre		
290	1307	2.5	116	523		The fretwork of *shesh tond-e-zamine dah*	Khayyatha Mosque
7775	9307	2.5	311	3723			
240	366	6	40	61			
755.2	1512	8	94.4	189			
2008	10776	8	251	1347			
1024	704	32	32	22			
1632	7344	24	68	306			
4368	52320	24	182	2180			
1702	2664	74	23	36			
408	822	6	68	137			
1092	5850	6	182	975			
322	224	14	23	16			
840	1700	20	42	85			
2240	12040	20	112	602			
1008	700	70	14.4	10			
2-A-4 workforce in Figure 3925=840+408+1632+755+290=			Workforce= frequency * Area				
2-A-5 workforce in Figure 10485.5=2240+1092+4368+2008+777.5=							
2-A-6 workforce in Figure 4296=1008+322+1702+1024+240 =							

Frequency * Perimeter	Frequency * Area (cm^2)	Tile color	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	mohre	The fretwork of *shesh tond-e-zamine kashi*	Khayyatha Mosque
1080	1444	White	38	27	38			
	76	Dark Blue	2					
6300	8520	Turquoise	284	21	30			
	480	White	16					
3496	2128	Ochre	304	11.5	7			
4884	2970	Dark Blue	3/2 (148)	33	30			
	1470	White	3/1 (148)					
1102	2394	-	38	29	63			
2-B-3 workforce in Figure 16862=1102+4884+3496+6300+1080=				Workforce=frequency * Area				
Amount of turquoise tiles 8520= (cm^2)				The amount of tiles used (in $cm^{2)}$ in different colors				
Amount of dark-blue tiles 3046=2970+76= (cm^2)								
Amount of white tiles3394=1470+480+1444= (cm^2)								
Amount of ochre tiles2128= (cm^2)								

of western side of the middle ivan of Khayyatha mosque(Fig. 4-a).The area, perimeter and frequency of individual shapes used in this fretwork have been given and then the workforce needed was calculated. In addition, the number of tiles used for each specific color was calculated (Table 4).

Fretwork of Kond-e-Sorme Dan Roo Alat

This fretwork, in Khayyatha mosque, has been accomplished totally in brick(Fig. 4-a).

The fretwork was analyzed into its constituents and the specifications of each were given. The amount of workforce was also calculated. In addition, considering the frequency and the area of each mohre, the number of tiles used for each specific color was calculated (Table 4).

The fretwork of Gol sabounaki

The examination of Persian tile work patterns indicates that many of them have been formed by cutting and arranging mono-color squared tile pieces (Sarhangi, 2008);(Fig. 5).

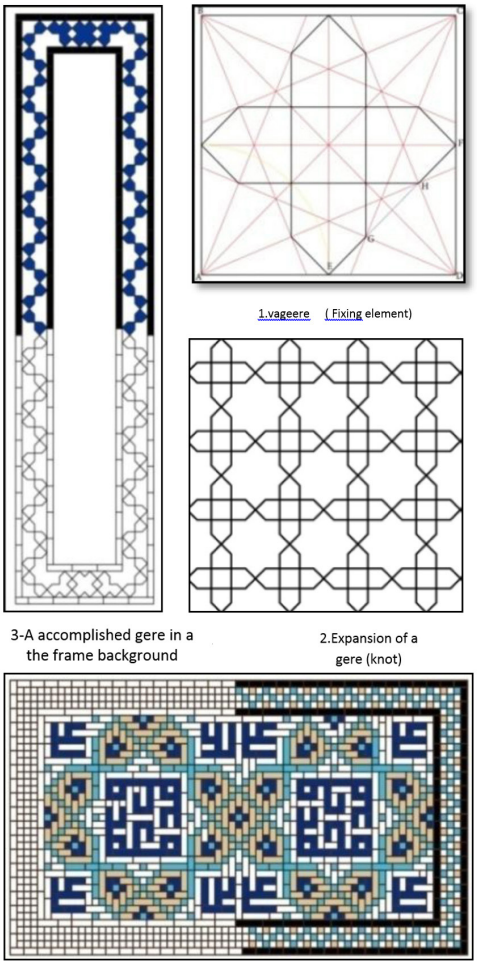
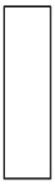
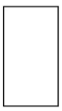
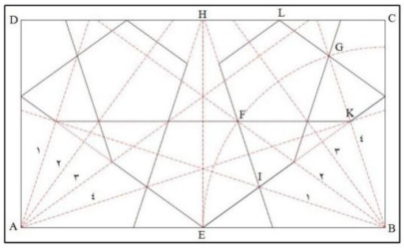


Fig. 3. The fretwork of Shamse-va-Chahar Soli-e-Moraba-Dar. Source: authors.

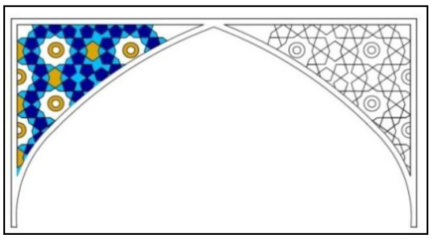
Table 3.The geometric specifications applied in the fretworks of Shamse-va-Chahar Soli-e-Moraba-Dar. Source: authors.

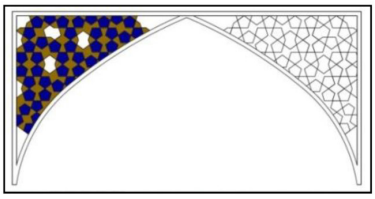
Frequency × perimeter	Frequency × Area (cm^2)	Tile Color	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre	
575	760	White	23	25	33		Neemavard Mosque
864	540	Dark Blue	108	8	5		
156	65	White	26	6	2.5		
The amount of workforce in Figure3-3 1595=156+864+575=				Workforce= frequency * Area			
540= The amount of dark-blue tiles in (cm^2)				The amount of tiles used (in $cm^{2)}$ in different colors			
825 = 65+760= The amount of white tiles in (cm^2)							

Frequency * Perimeter	Frequency * Area (cm^2)	material	Tile Color	Frequency in Frame	Perimeter (cm)	Area (cm^2)	Mohre	
2209	88	Brick	-	4	23.5	22		Neemavard Mosque
	572	Tile	Dark Blue	26				
	748		White	34				
	132		Turquoise	6				
	528		Black	24				
3002	759	Brick	-	46	19	16.5		
	429	Tile	Dark Blue	26				
	858		White	52				
	231		Turquoise	14				
	330		Black	20				
4116	1012	Brick	-	92	14	11		
	638	Tile	Dark Blue	58				
	1210		White	110				
	374		Turquoise	34				
9595	132	Brick	-	24	9.5	5.5		
	473	Tile	Dark Blue	86				
	1826		White	332				
	2596		Turquoise	472				
	528		Ochre	96				
The amount of workforce in Figure 4-3 18922=9595+4116+3002+2209=				Workforce= frequency * Area				
The amount of dark-blue tiles in (cm^2) 2112=473+638+429+572=				The amount of tiles used (in $cm^{2)}$ in different colors =				
tiles in (cm^2) The amount of white 4642 = 1826+1210+858+748=								
The amount of Turquoise tiles in (cm^2) 3333=2596+231+374+132=								
858=330+528= The amount of black tiles in (cm^2)								



1.vageere (Fixing element)





3-The accomplished gere in a spandrel frame in color

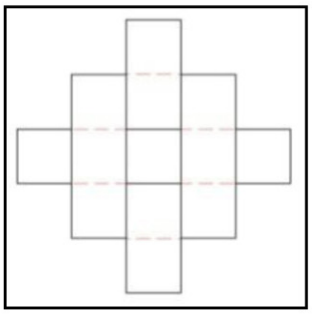
B- The fretwork *kond-e-sorme dan roo alat*

Fig. 4. the fretwork kond-e-sorme dan. Source: authors.

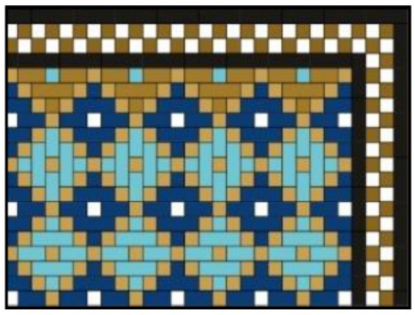
Table 4.The geometric specifications applied in the fretworks of Kond-e-Sorme. Source: authors.

Frequency * perimeter	Frequency * Area (cm^2)	Tile Color	Frequency in Spandrel	Perimeter (cm)	Area (cm^2)	Mohre		
45	123	White	1	45	123		The fretwork of kond-e-sorme dan ghenas koochak	Khayyatha Mosque
1296	1584	Dark Blue	72	18	22			
408	272	Ochre	34	12	8			
184	272	White	8	23	34			
1008	980	Ochre	28	36	35			
The amount of workforce in Figure 4-a-3 2934=1008+184+408+1296+44.5=			Workforce= frequency * Area					
The amount of dark-blue tiles (cm^2) 1584= tiles (cm^2) The amount of white 395=272+123= tiles (cm^2) The amount of ochre 1252=980+272=			The amount of tiles used (in $cm^{2)}$ in different colors =					

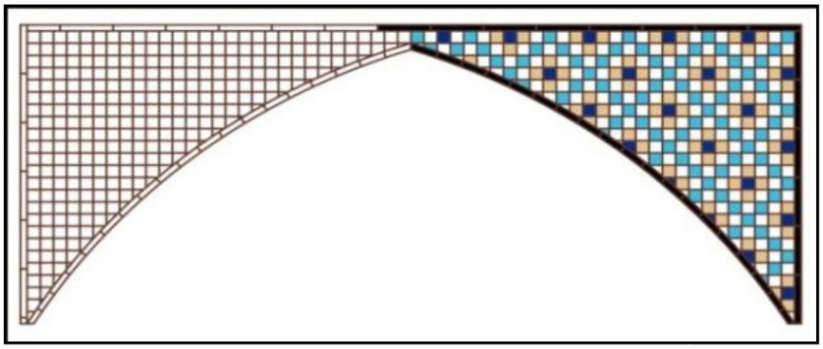
Frequency * Perimeter	Frequency * Area (cm^2)	Tile Color	Frequency in Spandrel	Perimeter (cm)	Area (cm^2)	Mohre	The fretwork *kond-e-sorme dan roo alat*	Khayyatha Mosque
427.5	1269	white	9	47.5	141			
912	1200	dark-blue	48	19	25			
1107	765	Turquoise	90	12.3	8.5			
171.5	266	ochre	7	24.5	38			
304	316	Turquoise	8	38	39.5			
The amount of workforce in Figure 4-B-2 2922=304+171.5+1107+912+427.5=			Workforce= frequency * Area					
tiles (cm^2) The amount of Turquoise 1081=316+765= 1200= The amount of dark-blue tiles 1269= tiles (cm^2) The amount of white 266= tiles (cm^2) The amount of ochre			The amount of tiles used (in cm^2) in different colors =					



vageere (Fixing element)



The accomplished fretwork in Khayyatha mosque



The accomplished fretwork in Nimavard mosque

Fig.5. The fretwork of Gol sabounaki. Source: authors.

Table 5.The geometric specifications applied in the fretworks of Gol sabounaki. Source: authors.

Frequency * Perimeter	Frequency * Area (cm^2)	Tile Color	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre	
3291	748	Dark-Blue	110	136	8.6		Khayyatha Mosque
	150	Ochre	22				
	748	Turquoise	110				
2640	935	Dark-Blue	110	10	5.8		
	374	Ochre	44				
	935	Turquoise	110				
2786	319	Turquoise	110	6.8	2.9		
	127.6	Ochre	44				
	348	Dark-Blue	120				
	174	White	60				
The amount of workforce in Figure 5-2 8717=2786+2640+3291=			Workforce= frequency * Area				
tiles (cm^2) The amount of Turquoise 2002=319+935+748= The amount of dark-blue tiles 2031=348+935+748= 174= The amount of white tiles (cm^2) The amount of ochre tiles (cm^2) 651=127+374+150=			The amount of tiles used (in $cm^{2)}$ in different colors =				

Frequency* Perimeter	Frequency* Area (cm^2)	Tile Color	Frequency in the Spandrel	Perimeter (cm)	Area (cm^2)	Mohre	
5520	1680	White	240	10	7		Nimavard Mosque
	1022	Turquoise	146				
	280	Dark-Blue	40				
	882	Ochre	126				
5520= The amount of workforce in Figure 5-3			Workforce= frequency * Area				
1022= tiles (cm^2) The amount of Turquoise 882= The amount of ochre tiles (cm^2) 280=(cm^2) The amount of dark-blue tiles 1680 = (cm^2) The amount of white tiles			The amount of tiles used (in $cm^{2)}$ in different colors =				

Table 6.The geometric specifications applied in the fretworks of Peeli and Moraba (Squared). Source: authors.

Frequency * Perimeter	Frequency * Area	Tile Color	Frequency in Frame	Perimeter (cm)	Area (cm^2)	Mohre	
700	385	Ochre	10	35	38.5		Neemavard Mosque
	385	Turquoise	10				
560	276	Ochre	8	35	34.5		
	276	Turquoise	8				
80	40	White	9.5	8.2	4.2		
144	148.5	White	9	16	16.5		
The amount of workforce in Figure 6 1484=144+80+560+700=			Workforce= frequency * Area				
661=276+385= The amount of ochre tiles 1525 = 1485+40= The amount of white tiles 661=276+385= The amount of turquoise tiles			The amount of tiles used (in $cm^{2)}$ in different colors =				

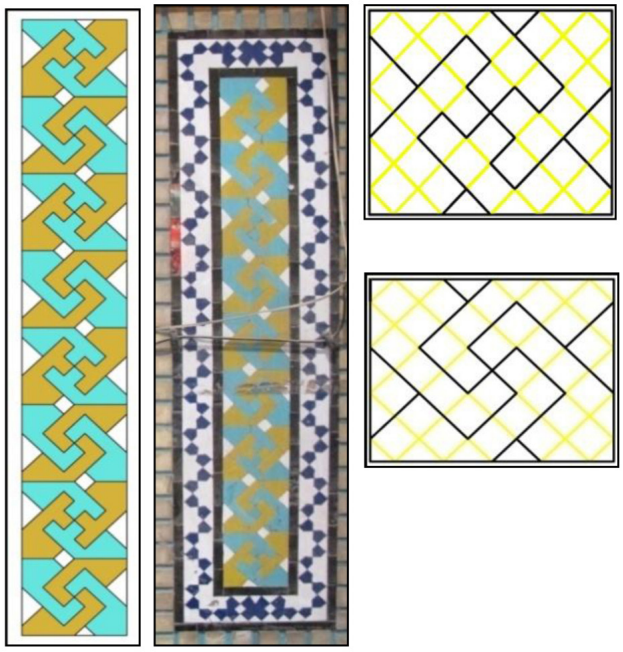


Fig.6. The fretwork of Peeli and Moraba. Source: authors.

The specifications of each form (area, perimeter and workforce) were calculated. Then, workforce used for each specific color was calculated (Table 5).

The fretwork of Peeli and Moraba (Squared)

This fretwork has been accomplished, totally in Nimavard mosque

Drawing Method:

A square with desired sides is drawn and each side is divided into eight equal parts. The division points of the adjacent sides are connected to each other so that a checkered slanted (45°) square is formed. The old lines in figures 1-6 represent vageere. The resulted form can be expanded through rotation (Maher-o-Naghsh, 1983: V.2, 116).

The fretwork was decomposed into its modules and the specifications of each (area, perimeter and workforce) were examined.The specifications of each form (area, perimeter and workforce) were calculated. Then, workforce used for each specific color was calculated (Table 6).

Discussion

Based on mentioned examinations and considering mathematical and geometric proportions, it was made possible to identify the characteristics of the tile fretworks applied and the amount of work force to accomplish them. Also, it was revealed that mathematical and geometric proportions could directly impact upon on the technique adopted to accomplish the fretworks and on the amount of work force needed.

Conclusion

The present study aimed to examine the decorative tile works applied in two mosques in Esfahan: Khayyatha mosque and Nimavard mosque. Also, the most frequently used fretwork and their formal characteristics were identified. The results showed that the pattern kond-e- dow panj was mainly applied in the spandrel of the main ivans (1-4). In addition, the patterns Gol sabounaki as well as shesh tond-e-zamine dah, were among the most commonly used fretworks in the two mosques (2-4 and 7-4) (Question 2).. The items which determine the amount of work force needed include the frame scale in which the fretwork has been applied; fretwork form as well as the geometric characteristics. Each of the fretwork patterns can be decomposed to smaller units. By examining the geometric characteristics of these forms, the amount of workforce needed can be calculated(Question 1). Among the studied fretworks, shamse and squared shahar soli had the highest workforce (the reason seems to be that the squared shahar soli has rectangular modules and, as a result, making them is rather a time-consuming process) (4-4). The second rank, in terms of workforce needed, goes to the fretworks Gol sabounaki and shesh tond-e-zamine hasht (4-3 and 4-7). Naturally, the more the amount of workforce is, the more time, cost and materials should be allocated (Question 3).

Persian architects always have done their best to avoid vanity and to make the most of materials. Based on the results obtained here it is possible to choose the best economical approach to apply tiles is adopted. Calculation of the amount of workforce not only may help us to have a closer estimation of costs but also may help the architect to order needed tiles based on desired color. All this, finally, leads to optimization of material consumption (Estimates of the labor force tiled intelligence on Iranian architect is a good proof).

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