

Persian translation of this paper entitled:  
بازخوانی تناسب هندسی در عناصر و اجزای ارتفاعی  
مسجد-مدرسه‌های دوره قاجار در شهر تهران  
published in this issue of journal

## Original Research Article

# Investigating the Geometrical Proportions in the Height Elements and Components of Qajar Era Mosque-Schools in Tehran

Hamed Hayaty\*

Assistant Professor, Architecture Department, Qom University, Qom, Iran.

Received: 05/07/2023 ;

accepted: 13/05/2024 ;

available online: 21/05/2024

## Abstract

**Problem statement:** geometry, along with proportions, are the significant indicators of the prominent monuments designed by Iranian artists and architects over the history of architecture in Iran. Architectural proportions create a desirable connection between the generating components of a monument and provide the audience with a sense of order and arrangement between different parts of a visual composition. Besides order, proportions can create a sense of beauty. In other words, Iranian architects apply geometry and proportions as a common language to meet the aesthetic, structural, and functional needs of the architecture and artworks.

**Research objective:** The current research explored the geometrical proportions in the height components of the Qajar Era school mosques in Tehran, Iran. Thus, each school mosque was analyzed based on the proportional rules of the height components. As a result, a height pattern or a relational pattern between the height components of 9 school mosques was obtained. Hence, a height design hierarchy can be reread in each mosque. Therefore, the research question was what common patterns and proportions can be found in the height components (dome, minarets, porches, courtyard facade) of the school mosques.

**Research method:** The current paper used a quasi-experimental research method to examine the compatibility between the dimensions of the height components and the proportions and geometry. Then, the deductive method was utilized to investigate the case studies. That is to say, after analyzing the data of each building and comparing them with the proportion principles, the characteristics of the common height proportions were observed in the height of the components of the building courtyards based on which the research findings were assessed.

**Conclusion:** After analyzing the proportions and height patterns in 9 case studies, the Iranian golden ratio was obtained in all the height components, including the height of the courtyard facade, from the shortest height of the porch to the highest minaret or dome. The common patterns in the proportions were found in all case studies while comparing the height components. For instance, the shortest porch of the complex and the height of the courtyard facade had proportions close to the golden ratio. Furthermore, other similar proportional patterns were recognized in two or several school mosques.

**Keywords:** *Golden ratio, Iranian proportions, height components, Qajar era, Tehran mosques.*

## Introduction

Since the creation of humans and their awareness of their surrounding environment, the use of proportions has always been a matter of question

and analyzed by various artists and scientists (Kurt Grütter, 1987). Proportions have always played a role in the architecture of Iran and the world, and in the past architects have used mathematics and geometry to create proportion and harmony in their

\*Corresponding author: hamedhayaty@yahoo.com, +989189898190

works (Khavas & Bemanian, 2022, 2). By using proportions and geometry, architects have historically succeeded in creating order, harmony, hierarchy, and aesthetics in a series of adjacent buildings with different land uses (Lu & Steinhardt, 2007). Proportions can be considered a way of mathematical connection between the components of a monument, creating order and visual aesthetics in an artistic complex (Doczi, 1981). The current research explored the trace of geometrical proportions in the height components and their emergence in mosques. As a type of prominent urban building in every historical period, religious buildings can be considered an example with geometrical and arithmetic ratios designed and built by the architects of that specific period. Moreover, these buildings have always been an attention center, assessment, and evolution in architecture. The current study attempted to analyze and examine the height proportions and their impact on the architectural prosperity of the school mosques. Also, by analyzing and assessing the height proportions of a school mosque, one can find a specific height pattern for a school mosque and consider a certain height design hierarchy for each case. As a result, a height hierarchy can be considered for each building by investigating how the dimensions and height of the school-mosque components are determined by an architect or the building's evolution during different historical periods. In other words, it can be determined how, in a height hierarchy, each component has been placed in the building based on the relationship between that component and the previous component with specific proportions. Finally, the common patterns of the different school mosques can be extracted. Hence, the main research question is what common proportions or patterns can be found between the height components (dome, minarets, porches, and courtyard facade) of the Qajar Era school mosques in Tehran, Iran.

### Research Background

The research background was limited to analyzing the architectural plan characteristics and lacked attention to the height components in the global and national research. Also, investigating the height of the

architectural components was constrained to the few prominent mosques, the important of which are listed as follows:

The research entitled "Courtyard Proportion of Safavid Era Mosques in Isfahan" by Attarian et al. (2016) explored the changes and characteristics of the proportions in the yards of the Safavid Mosques to address the ratio(s) between the sides of the yards in mosques. "Geometrical Patterns and Golden Ratios; A Common Architectural Language in Abyaneh, the Historical Village" (Balilan & Hassanpour Lamar, 2019) attempted to find out how the identical geometrical patterns have been formed in such a vast level of artistic motifs by identifying the geometrical patterns in all artistic motifs of Abyaneh. In "Compatible Study of the Facade Proportions of the Portal of Sheikh Lotfollah Mosque, Abbasi Mosque, Hakim Mosque, and Chahar Bagh School of Isfahan", Pourmand et al. (2014) analyzed the proportions of the portal of several cases from the prominent Safavid monuments in Isfahan. Dabbour (2012) discussed the geometrical ratios as a design tool for arabesque motifs in research entitled "Geometric proportions: The underlying structure of design process for arabesque geometric patterns." Moreover, the key role of geometry in the arabesque patterns was discussed regarding the proportions operated as the underlying structure of the design process. Analysis of the geometric principles and the design features of the arabesque-traditional patterns was also addressed, including the analysis of the geometric proportions. Dabbour described the components of the arabesque patterns in terms of Islamic Cosmology, philosophy, and metaphysics of geometry, which can be seen as subjection of a predetermined principle of geometric and proportional design with high accuracy and precision. In "Investigation the theory of proportions adjustment system in historical houses of Qajar period in Khorramabad" Naseri et al. (2020) attempted to reread the geometry, proportions, and maintaining them in contemporary architecture to find the proportion adjustment systems applied in the Qajar Houses of Khorram Abad. Shamaileh (2022), in "Study and investigate perceptions of Jordanian college students on

the golden ratio proportions”, focused on the importance of the golden ratio in the concepts of interior design. The results showed that the participants’ reactions were significant as they found the golden ratio-based forms more attractive and better than the normal forms. Then, the author suggested that the golden ratio should be more considered in interior design. The research entitled “Proportions in the height components of the four-porch mosques” (Khavas & Bemanian, 2022) explored the trace of proportions in the Iranian mosques as one of the most significant elements of Islamic Architecture in Iran. In “A Comparative Study of the Entrance Circulation of Abbasi Jame’ Mosque and Seyyed Jame’ Mosque in Isfahan from the Perspective of the Persian Geometric Proportion in an Architectural Plan,” Abbasi et al. (2020) explored the system of geometrical proportions in generating the components and relationships ruling the structure of the entrance plan. It is noteworthy that previous research addressed the architectural plans and lacked analyzing the height components and elements. Furthermore, the relationships between the proportions of different elements of school mosques of the Qajar era in Tehran have not been examined, which is the novelty of the current study.

**Research Method**

The current research applied the quasi-experimental research method to test the compatibility of the dimensions of height elements with the proportions and geometry. The samples were selected from the prominent and famous school mosques of Tehran built in the Qajar era using a non-random sampling method to have the highest geometrical diversity and height elements. The dimensions of school mosques were collected using library and survey methods. The deductive method was also applied to study and assess the samples. In other words, after analyzing the data of each building and comparing them with the principles of proportions, the characteristics of the common height proportions in the height of the elements of the building courtyards were observed, based on which research findings were evaluated. Evaluated variables of the proportions of the height elements were selected, and their proportions

were assessed compared to each other. These variables included the height of porches, the width of porches, the length and width of yards, the height of domes, the height of minarets, the height of courtyard facades, and the height of the stories of the courtyard facades. All the main height elements of the mosques, in addition to the components of the buildings, were observed in terms of visual interactions with the height components (Fig. 1).

**Theoretical Framework**

• **Proportions in architecture and Iranian proportion**

Proportion is one of the primary principles of an artwork, representing the harmony between the elements (Ansari et al., 2011, 46). According to Euclid, ratio refers to the quantitative comparison between two similar things, and proportion is the equality of the ratios. Proportions are a set of ratios. Ratio is the comparison between two qualities of quantities, such as amount or quantity, indicating a unit of difference (Krier, 1988, 7). While proportion is a determining factor for harmony, it has been one of the most controversial subjects in architecture. Proportion in architecture is the ratio, which represents the relationship between two or more quantities (Kurt Grütter, 1987, 360). The proportion system of Islamic architecture is based on the geometrical characteristics of square, addition square,

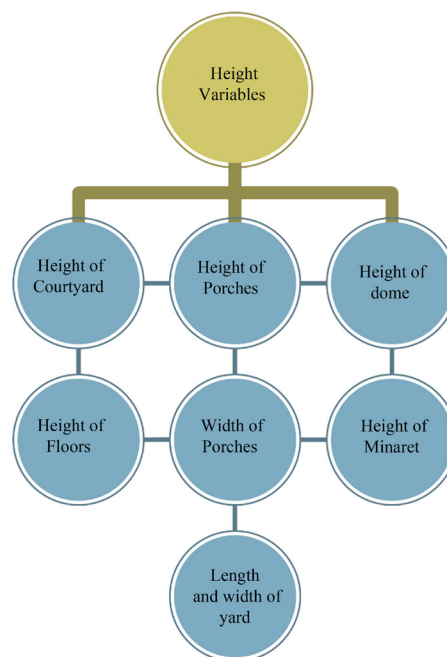


Fig. 1. Variables of height proportions. Source: Author.

equilateral triangle, and pentagon that equal irrational numbers (Golombek & Wilber, 1988, 194). Iranian proportions are  $\sqrt{2}$  and  $\sqrt{3}$ , which equals 1.41,  $\sqrt{3}$ , which equals 1.73,  $\sqrt{1.25}$ , which equals 1.118, and  $\sqrt{5.2}$ , which is taken from  $\sqrt{2}$  and  $\sqrt{3}$  (Bamanian et al., 2011, 139) (Fig. 2). As a result, to find the Iranian proportions in the structure of a building, these numbers must be found by comparing the architectural components of a building.

• **Golden proportions**

Pythagoras believed that events occur based on general principles. A principle can play a role in generating an artwork if it considers the order in the mathematical rules. The harmonic proportions is a general principle for applied and theoretical sciences. The golden proportion was first raised by Euclid in the 3rd Century BC. Later on, the Greeks found out the significant role of golden proportions in the human body’s proportions. By believing that humans and their temples must belong to a superior order of the world, they used the same proportions in building their temples. Architects also used the same rule in the Renaissance era (Doczi, 1981, 97). Euclid was the first person who described the golden proportions and their divisions precisely. Later on, this ratio was considered and applied as the ideal proportions for creating aesthetics and harmony in the elements of an artwork (Leopold, 2006).

The golden ratio is a special number in art, architecture, and design, which has been considered by artists since the past. Golden division refers to dividing a line into two parts: long part and short part. The entire length divided by the long part equals the long part divided by the short part, which equals 1.681 (Lu & Steinhardt, 2007, 1108). The golden ratio is also known as the phi number (Putz, 1995, 275) (Figs. 3 & 4).

Therefore, the theoretical framework of the current research included identifying the proportions in the height components and elements of the school mosques to obtain the hierarchy of determining the height dimensions of each component to other components for each monument and recognizing similar patterns.

• **Introducing school-mosques and analyzing the height proportions**

Nine historical school mosques of the Qajar era in

Tehran were purposefully selected to explore and recognize the geometric ratios based on the following reasons (Table 1):

- Having diverse height elements (domes, minarets, porches, courtyard facades, floor facades), which makes the selected mosques to be considered from various perspectives in terms of geometry.
- Being a historical monument, being intact, under study, and strong form of the height components in terms of geometric proportions.

**Discussion**

Variables or proportions of the reread height components were investigated, including the length and width of the yard, the height of porches, the height of the facade of the yard, the facade of the floors, the height of the dome, and the height of the minaret. Moreover, the ratio of each component was compared with other parts of the building, and patterns were obtained from the proportions based on the numbers and proportions provided in the theoretical framework. These patterns have been analyzed in the following paragraphs.

• **Imam mosque of Tehran**

The height of the eastern porch is equal to the western

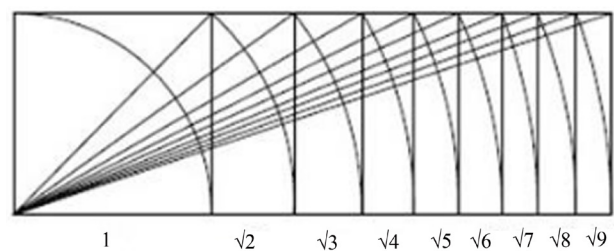


Fig. 2. Drawing  $\sqrt{2}$  rectangle and expanding it. Source: Buzjani, 1997, 125.

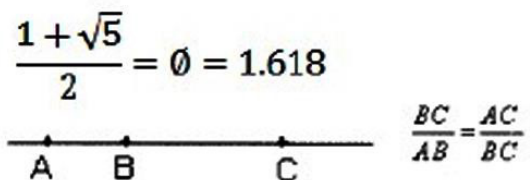


Fig. 3. Dividing a line using golden proportions. Source: Gangwar, 2017, 172.

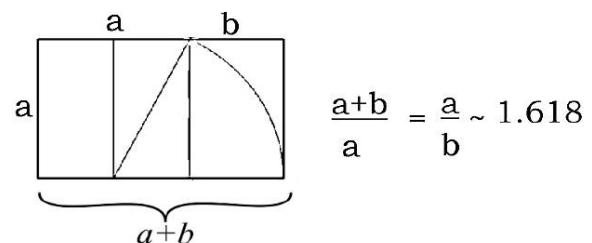


Fig. 4. Golden rectangle. Source: Balilan & Hasanpour Lamar, 2019, 52.

Table 1. The features of the studied cases. Source: Author.

Row	Name of School-mosque	Building founder and history	Building structure	Building Photo	3D image of the building
1	Imam Mosque	Qajar era- (1845)- Fath Ali Shah (Tayfeh, 2016, 17)	Four-porch with tile works and arching, three Shabistan, two minarets, square-shaped yard (Saatian & Moghaddasi, 2010, 32)		
2	Tehran Bazaar Central Mosque	Qajar era-(1861) Razi Alldin (Etemad Al-Saltaneh, 1876)	Two porches, four Shabistans, rectangle-shaped and two-story yard (Mostafavi, 1982)		
3	Hakim-Bashi Mosque	Qajar era- 1855-Mirza Ahmad Hakim Bashi (Hosseini Balaghi, 1971)	Three porches, one shallow shabestan, a relatively square-shaped yard, and one dome (Haji Ghasemi, 1998, 32)		
4	Sheikh Abdolhossein mosque and school	Qajar era- 1891- Sheikh Abdolhossein Tehrani (Hosseini Balaghi, 1971)	Four porches, two shabestan, a vast yard with two-story chambers, including a maqsurah, one dome, and two small minarets (Haji Ghasemi, 1998, 40)		
5	Shahid Motahhari School-mosque (Sepahsalar mosque)	Qajar era- 1919 Mirza Hossein Khan Sepahsalar (Haji Ghasemi, 1998, 92)	Four porches, square-shaped yard, including a maqsurah, a big dome, and 8 minarets (Afshar, 1965)		
6	Moayer al-Mamalik Mosque and Seminary	Qajar era- 1905 Doust Ali Khan Nezam Al-doleh (Etemad Al-Saltaneh, 1976)	Single porch, shallow Shabestan, a great dome house without ornaments, and two unfinished minarets (Zoka, 1967)		
7	Sepahsalar Mosque-School	Qajar era- 1904 Mirza Mohammad Khan Sepahsalar (Hosseini Balaghi, 1971)	Single porch, two shabestans, square-shaped yard, two minarets (Hosseini Balaghi, 1971)		
8	Filsouf – ol-Doleh School mosque	Qajar era- 18910 Mirza Kazem Tabib Rashti (Hosseini Balaghi, 1971)	Two porches, one shabestan with columns, two unfinished minarets, and 12 chambers (Haji Ghasemi, 1998, 126)		
9	Marvi School Mosque	Qajar era- 1853 Haj Mohammad Hossein Khan (Hosseini Balaghi, 1971)	Four porches, one shabestan, rectangle-shaped yard, cistern (Mostafavi, 1982)		

porch, and the height of the northern porch is equal to the height of the dome. Moreover, the width of the eastern and western porches follows the golden proportions. The height hierarchy in the mosque consists of the height of the whole courtyard having a 1.5 ratio to the eastern and western porches and twice the northern porches (Fig. 5). Then, the  $\sqrt{2}$  can be seen by comparing the height of the southern porch with the dome, and the height proportion of the northern porch is twice the minaret. Also, the highest points of the dome and the minaret have a ratio of  $\sqrt{2}$ . Thus, the height and width of the porches, dome, minaret, and courtyard have followed the Iranian and golden proportions (Table 2 & Fig. 6).

• **Jame mosque of Tehran**

The height and width of the eastern porch follow the golden proportions, while the height of the southern porch (the highest porch) is twice its width and twice the width of the eastern porch. The height hierarchy of the mosque shows

that the height of the courtyard facade has a proportion of 1.5 times the height of the facade of the second floor and close to the height of the first floor. On the other hand, there is a double proportion in comparing the height of the eastern porch with the height of the facade of the first floor (Fig. 7). Golden proportions can also be found in the comparison between the height and width of the eastern porch. As a result, the height and width of the southern and eastern porches, the facade of the courtyard, and the facade of the floors of the mosque follow the Iranian and golden proportions (Table 3 & Fig. 8).

• **Hakim Bashi mosque**

The height of the southern porch is twice the width of the eastern and western porches and  $\sqrt{2}$  is equal to the width of the porch. The height hierarchy of the mosque shows that the height of the facade of the courtyard has a proportion of 1.5 times the height of the eastern and western porches and is nearly twice the height of the southern porch.

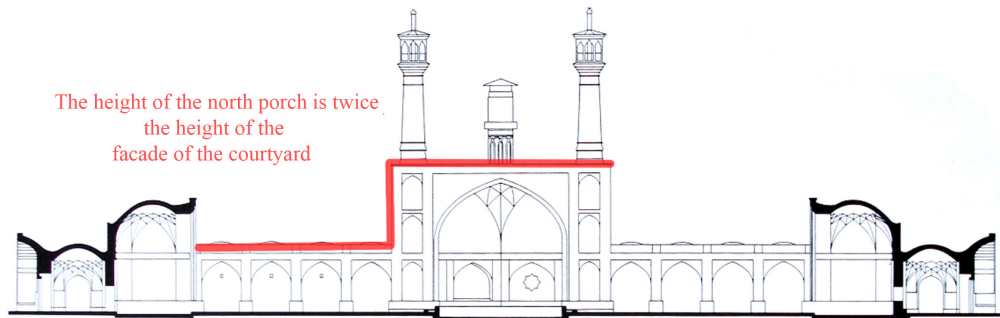


Fig. 5. The facade of the northern porch in Imam Mosque. Source: Haji Ghasemi, 1998, 19.

Table 2. The height proportions of the Imam Mosque in Tehran (Unit: Metric system). Source: Author.

Imam Mosque of Tehran	Dimensions	Height of the facade of the courtyard	Height of the northern porch	Height of the southern porch	Height of the eastern and western porches	Height of the minaret	Height of the dome
Dimensions	-	7/4	14/4	16/4	11/1	29/4	22/3
Height of the facade of the courtyard	7/4	1	1/94	2/21	1/5	3/97	3/01
Height of the northern porch	14/4	0/51	1	1/14	0/77	2/04	1/55
Height of the southern porch	16/4	0/45	0/88	1	0/67	1/79	1/36
Height of the eastern and western porches	11/1	0/66	1/29	1/48	1	2/65	2/01
Height of the minaret	29/4	0/25	0/49	0/56	0/38	1	0/76
Height of the dome	22/3	0/33	0/64	0/73	0/49	0/32	1
Length of the yard	65/5	0/11	0/22	0/25	0/17	0/45	0/34
Width of the yard	65/5	0/11	0/22	0/25	0/17	0/45	0/34
Width of the northern and southern porches	12/4	0/59	1/16	1/32	0/89	2/37	1/79
Width of the eastern and western porches	8/5	0/87	1/69	1/93	1/3	3/46	2/62

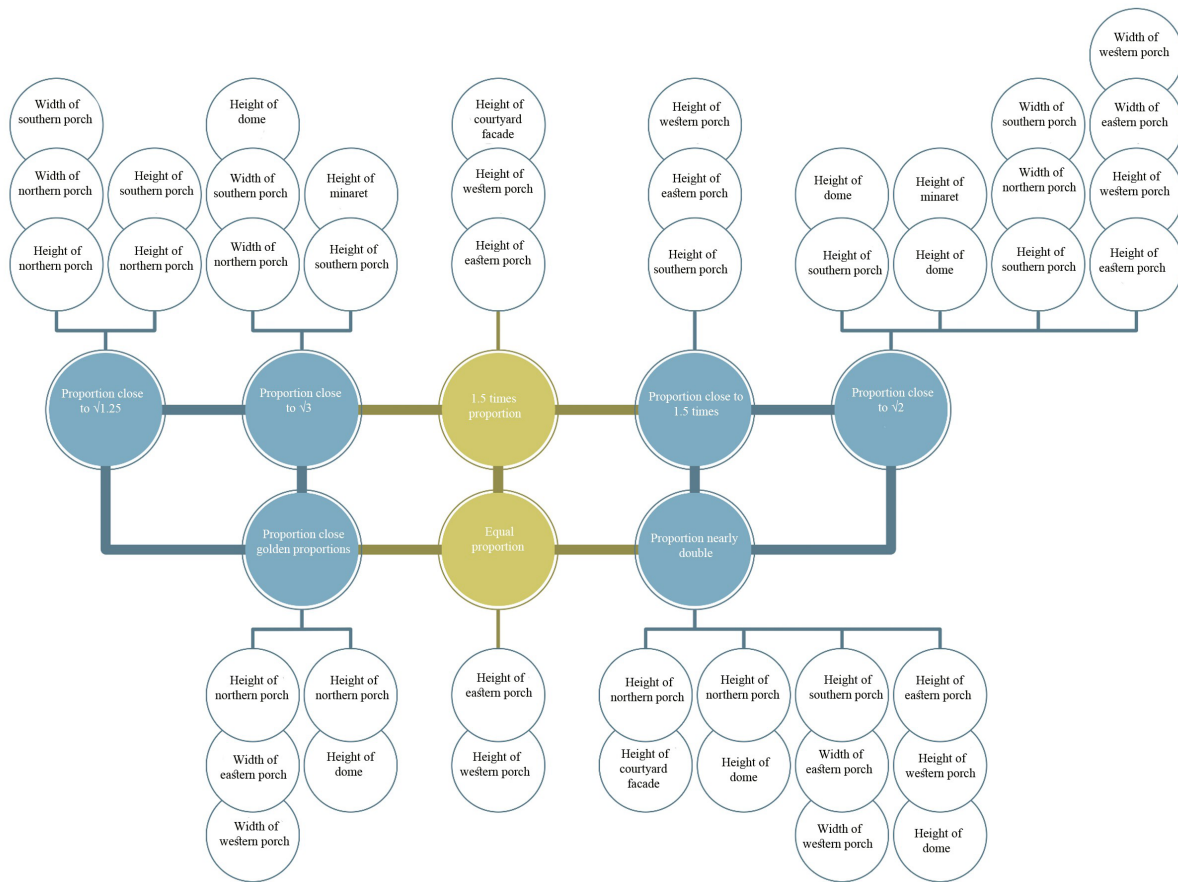


Fig. 6. Analyzing height proportions of Imam Mosque in Tehran. Source: Author.



Fig. 7. The facade of the east porch of Jame Mosque. Source: Haji Ghasemi, 1998, 29.

Table 3. The height proportions of Jame Mosque of Tehran ( Unit: metric system). Source: Author.

Jame Mosque of Tehran	Dimensions	Height of the facade of the courtyard	Height of the facade of the first floor	Height of the facade of the second floor	Height of the east porch	Height of the southern porch
Dimensions	-	4/5	4/7	3	9/9	12
Height of the facade of the courtyard	4/5	1	1/04	0/66	2/2	2/66
Height of the facade of the first floor	4/7	0/95	1	0/64	2/01	2/55
Height of the facade of the second floor	3	1/5	1/57	1	3/3	4
Height of the east porch	9/9	0/45	0/47	0/3	1	0/12
Height of the southern porch	12	0/37	0/39	0/25	0/82	1
Length of the yard	65/5	0/07	0/07	0/04	0/15	0/18
Width of the yard	22/3	0/2	0/21	0/13	0/44	0/54
Width of the east porch	6/2	0/2	0/76	0/58	1/59	1/93
Width of the southern porch	8/2	0/55	0/57	0/36	1/21	1/46

Furthermore, there is a proportion of  $\sqrt{2}$  in the comparison between the height of the southern porch and the height of the eastern and western porches. Finally, the dome has a proportion of  $\sqrt{3}$  in comparison with the height of the eastern and western porches, and a proportion of  $\sqrt{2}$  in comparison with the height of the southern porch (Fig. 9). As a result, the height and width of the southern, eastern, and western porches and the dome of the mosque follow the Iranian proportions (Table 4 & Fig. 10).

• **Sheikh Abdolhossein school mosque of Tehran**

The height hierarchy of the mosque shows that the height of the facade of the courtyard has a proportion

close to twice the height of the four porches, and a proportion of  $\sqrt{1.25}$  the height of the facade of the first floor (Fig. 11). The height of the northern porch is equal to the height of the southern porch, and the height of the eastern porch is equal to the height of the western porch. Moreover, they have a proportion of  $\sqrt{1.25}$ . The northern and southern porches have a proportion of 1.5 times the dome, and the eastern and western porches have a proportion of  $\sqrt{2}$  the dome. Furthermore, there is a proportion close to the golden proportions in comparison with the height of the minaret and the southern and northern porches, while

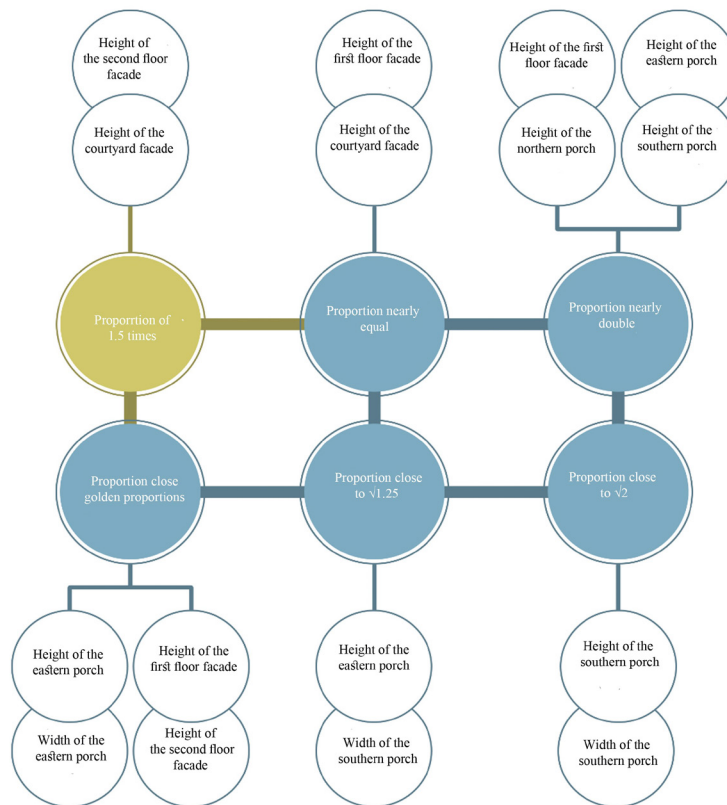


Fig. 8. Analyzing different height proportions of Jame Mosque of Tehran. Source: Author.

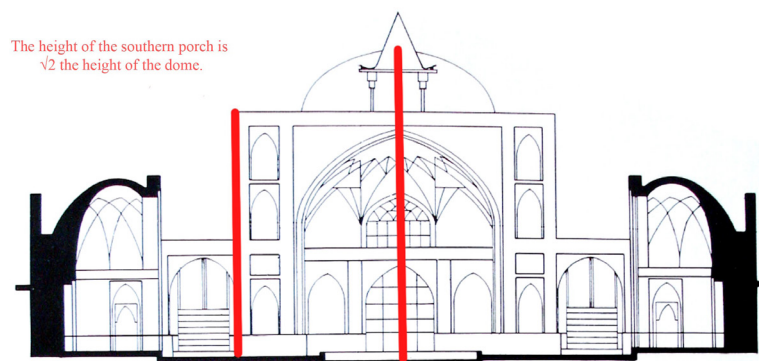


Fig. 9. The view of the southern porch in Hakim Bashi Mosque. Source: Haji Ghasemi, 1998, 38.

Table 4. Height proportions of the Hakim Bashi Mosque in Tehran ( Unit: metric systems). Source: Author.

Hakim Bashi Mosque of Tehran	Dimensions	Height of the facade of the courtyard	Height of the southern porch	Height of the eastern and western porches	Height of the dome
Dimensions	-	5	10/2	7/5	13/6
Height of the facade of the courtyard	5	1	2/04	1/5	2/72
Height of the southern porch	10/2	0/49	1	0/73	1/33
Height of the eastern and western porches	7/5	0/66	1/36	1	1/81
Height of dome	13/6	0/37	0/75	0/55	1
Length of the yard	23/7	0/21	0/43	0/31	0/57
Width of the yard	18/9	0/26	0/54	0/39	0/72
Width of the southern porch	7/5	0/66	136	1	1/81
Width of the eastern and western porches	5/2	0/96	1/96	1/44	2/61

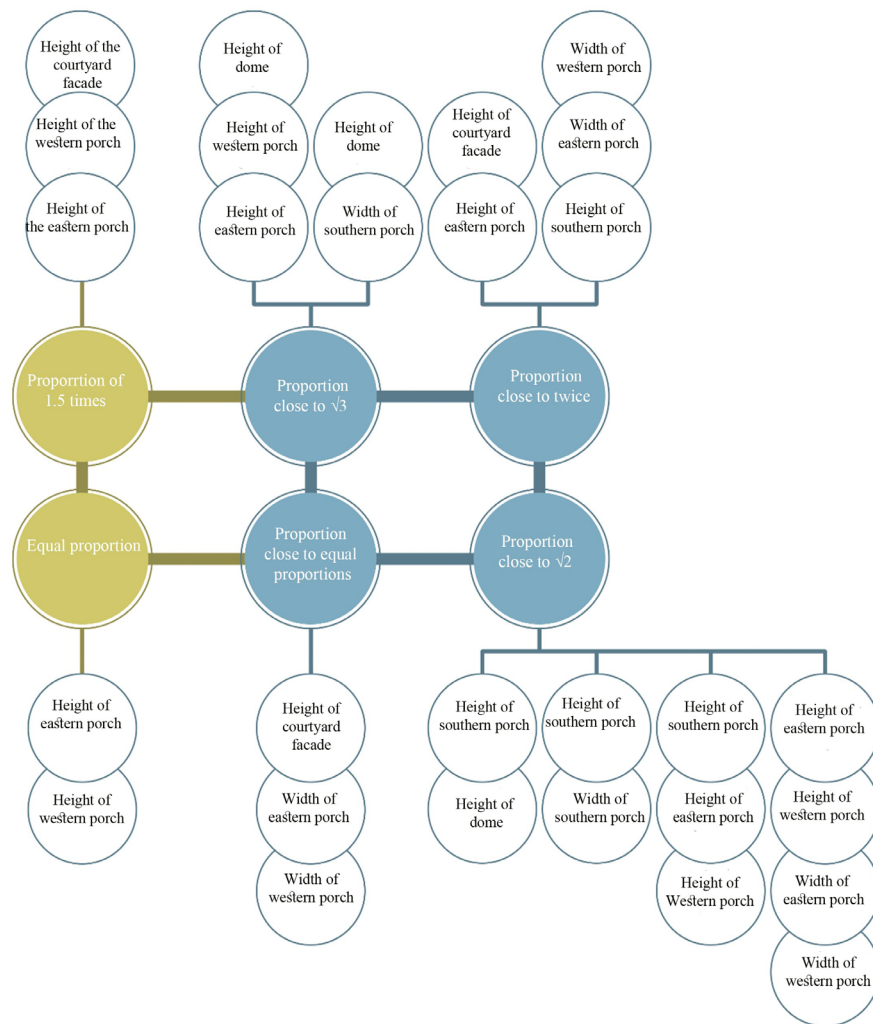


Fig. 10. Analysis of the height proportions of Hakim Bashi Mosque in Tehran. Source: Author.

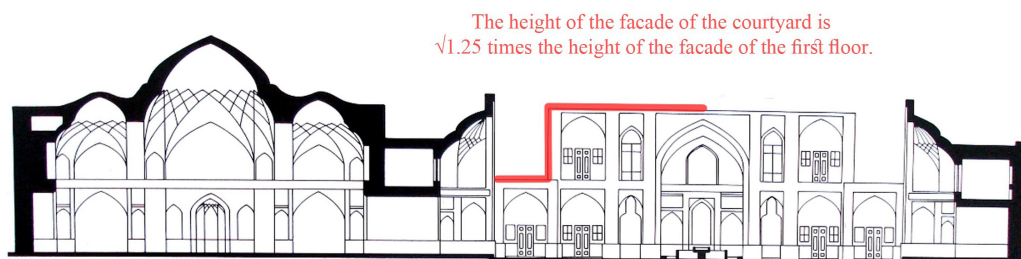


Fig. 11. The facade of the southern porch and Sheikh Abdolhossein mosque. Source: Haji Ghasemi, 1998, 46.

there is a proportion close to  $\sqrt{2}$  in the comparison between the height of the minaret and the eastern and western porches. Finally, a proportion of  $\sqrt{1.25}$  was found between the minaret and the dome. As a result, all the height elements of this mosque follow the golden and Iranian proportions (Table 5 & Fig. 12).

• **Shahid Motahari school mosque (Sepahsalar)**

The height hierarchy of the mosque shows that the height of the facade of the courtyard has a proportion of  $\sqrt{2}$  times the height of the eastern and western porches and a proportion of  $\sqrt{3}$  times the height of the southern porch. The western, eastern, and northern porches have

Table 5. The height proportions of Sheikh Abdolhossein school mosque of Tehran (Unit: metric system). Source: Author.

Sheikh Abdolhossein School Mosque	Dimensions	Height of the facade of the courtyard	Height of the facade of the first floor	Height of the northern and southern porch	Height of the eastern and western porch	Height of the minaret	Height of the dome
Dimensions	-	5/7	5	10/5	11/5	16/5	15/5
Height of the facade of the courtyard	5/7	1	0/88	1/94	2/02	2/89	2/72
Height of the facade of the first floor	5	1/14	1	2/1	2/3	3/3	3/1
Height of the southern and northern porches	10/5	0/54	0/47	1	1/09	1/57	1/47
Height of the western and eastern porches	11/5	0/49	0/43	0/91	1	1/43	1/35
Height of the minaret	16/5	0/34	0/3	0/63	0/69	1	0/94
Height of the dome	15/5	0/37	0/32	0/68	0/74	1/06	1
Length of the yard	37/6	0/15	0/13	0/28	0/3	0/44	0/41
Width of the yard	28/8	0/19	0/17	0/36	0/4	0/57	0/54
Width of the northern and southern porches	6/5	0/87	0/78	1/61	1/77	2/54	2/38
Width of the eastern and western porches	5/8	0/98	0/86	1/81	1/98	2/84	2/67

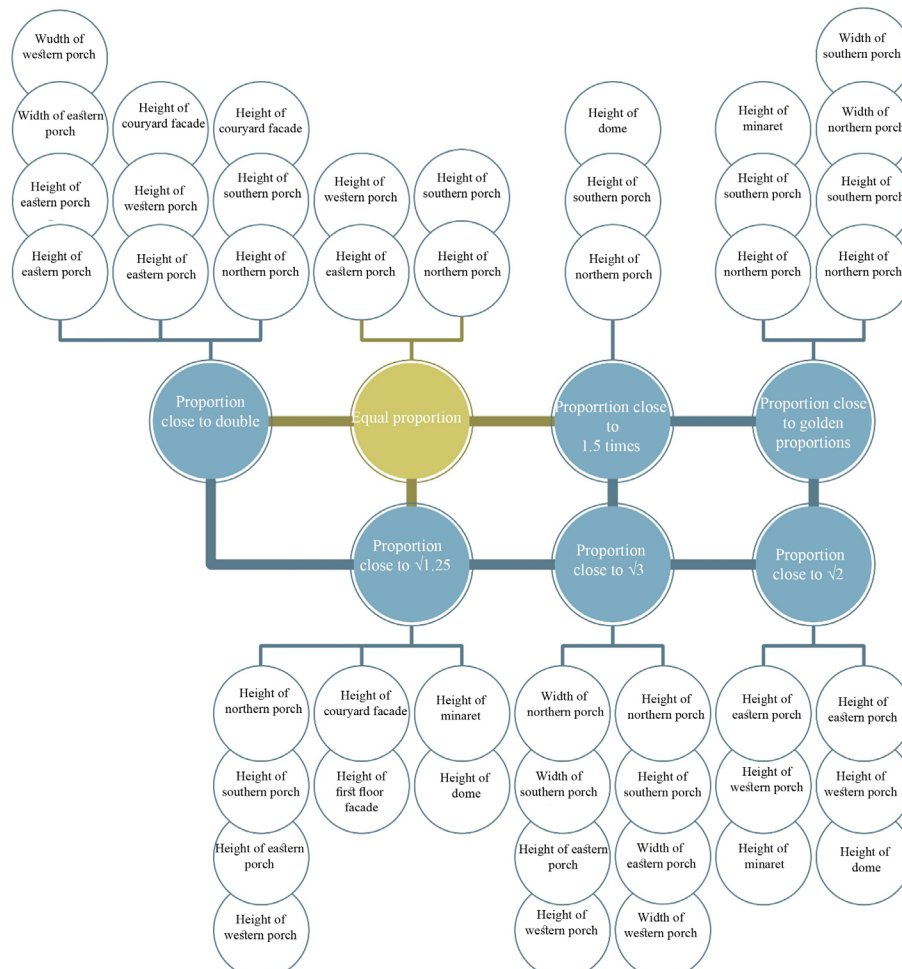


Fig. 12. Analysis of the height proportions of Sheikh Abdolhossein school mosque of Tehran. Source: Author.

equal height, and a proportion of  $\sqrt{2}$  times the southern porch. Moreover, the height of the dome is nearly twice the height of the northern, eastern, and western porches, and  $\sqrt{2}$  times the southern porch. The highest minaret has a proportion of  $\sqrt{3}$  times the highest porch, which is the southern porch. Thus, the height and width of the porches, dome, minaret, and facade of the courtyard of the mosque follow the Iranian and golden proportions (Table 6 & Figs. 13 & 14).

• **Moayer al-Mamalik mosque and seminary**

The height hierarchy in the mosque shows that the height of the facade of the courtyard has a proportion of twice the height of the facade of the first floor and a proportion of close to the golden proportions compared with the southern porch. The height of the southern porch has a proportion of twice the dome (Fig. 15) and a proportion of  $\sqrt{3}$  times the minaret. Finally, the minaret and dome have a proportion of  $\sqrt{1.25}$ . As a result, the height and width of the southern porch, dome, minaret, and facade

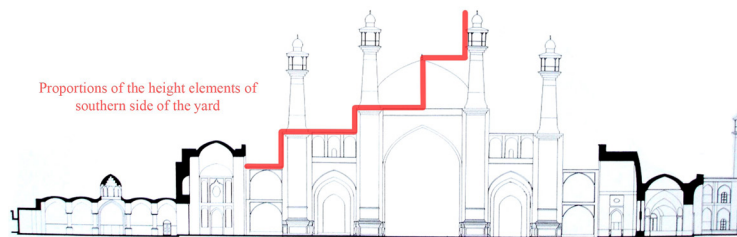


Fig. 13. Facade of the southern porch of the Shahid Motahari School Mosque. Source: Haji Ghasemi, 1998, 99.

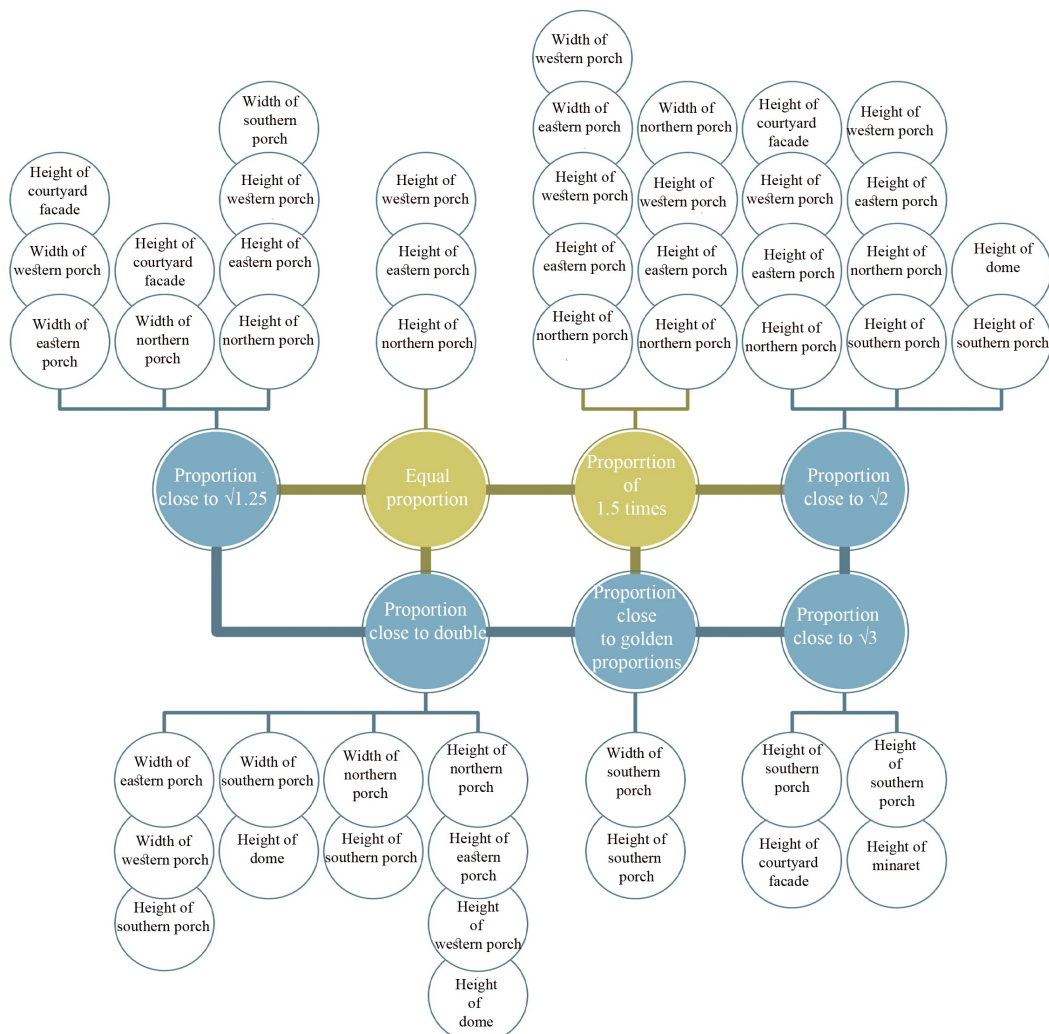


Fig. 14. Analysis of the height proportions of Shahid Motahari School-Mosque of Tehran. Source: Author.

of the courtyard of the mosque follow the Iranian and golden proportions (Table 7 & Fig. 16).

• Sepahsalar school mosque

The width of the northern and southern porches and the height of the facade of the courtyard show

the golden proportions. The height hierarchy in the mosque indicates that the height of the facade of the courtyard has a proportion of  $\sqrt{1.25}$  times the height of the northern and southern porches (Fig. 17). Moreover, the height of both porches is equal

Table 6. Height proportions of Shahid Motahari School Mosque of Tehran (Unit: Metric system). Source: Author.

Shahid Motahari School mosque of Tehran	Dimensions	Height of the facade of the courtyard	Height of the facade of the first floor	Height of the northern porch	Height of the southern porch	Height of the eastern and western porches	Height of the dome	Height of the minaret
Dimensions	-	14/5	6/6	18/8	26/6	18/8	36/8	46/4
Height of the facade of the courtyard	14/5	1	0/45	1/3	1/83	1/3	2/54	3/2
Height of the facade of the first floor	6/6	2/19	1	2/85	4/03	2/85	5/57	7/06
Height of the northern porch	18/8	0/77	0/35	1	1/41	1	1/96	2/47
Height of the southern porch	26/6	0/54	0/25	0/7	1	1	1/38	1/74
Height of the eastern and western porches	18/8	0/77	0/35	1	1/41	1	1/96	2/47
Height of dome	36/8	0/39	0/18	0/51	0/72	0/51	1	1/26
Height of minaret	46/4	0/31	0/14	0/4	0/57	0/4	0/79	1
Length of yard	80	0/18	0/08	0/23	0/33	0/23	0/46	0/58
Width of yard	80	0/18	0/08	0/23	0/33	0/23	0/46	0/58
Width of the northern porch	12/5	1/16	0/53	1/5	2/13	1/5	2/94	3/74
Width of the southern porch	17/4	0/83	0/89	1/08	1/53	1/08	2/11	2/66
Width of the eastern and western porches	12/5	1/16	0/53	1/5	2/13	1/5	2/94	3/74

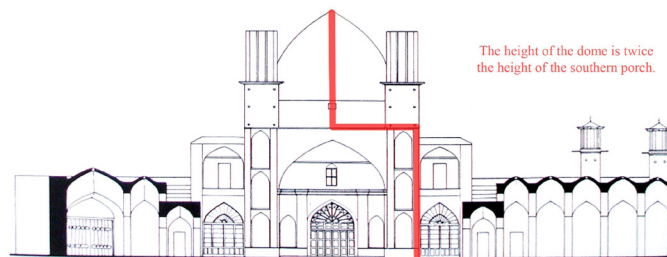


Fig. 15. The facade of the southern porch of Moayer Al-Mamalik Mosque and Seminary. Source: Haji Ghasemi, 1998, 107.

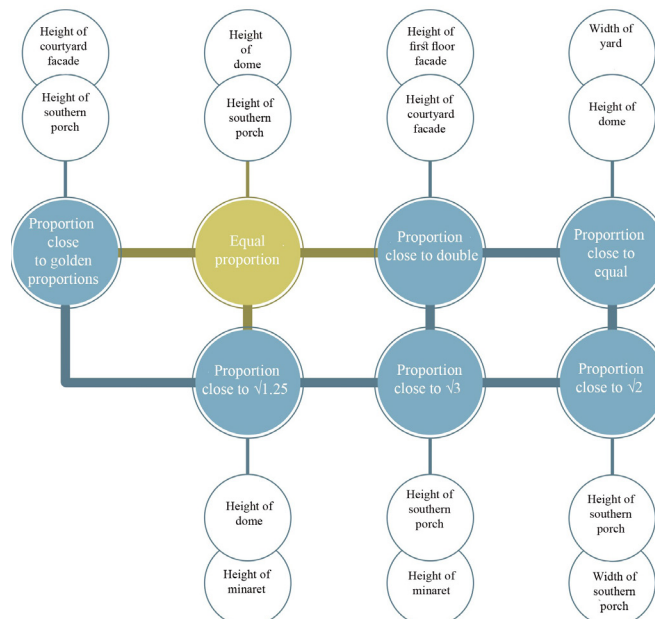


Fig. 16. Analysis of the height proportions of Moayer Al-Mamalik Mosque and Seminary of Tehran. Source: Author.

Table 7. Height proportions of Moayer Al-Mamalik Mosque and Seminary of Tehran (Unit: Metric system). Source: Author.

Moayer Al-mamalki mosque and seminary	Dimensions	Height of the facade of the courtyard	Height of the facade of the first floor	Height of the southern porch	Height of the dome	Height of the minaret
Dimensions	-	6/2	3	10/5	21	19
Height of the facade of the courtyard	6/2	1	0/48	1/69	3/39	3/06
Height of the facade of the first floor	3	2/06	1	3/5	7	6/33
Height of the southern porch	10/5	0/59	0/3	1	2	1/81
Height of the dome	21	0/29	0/14	0/5	1	0/9
Length of yard	34/8	0/18	0/08	0/3	0/6	0/54
Width of yard	21/1	0/29	0/14	0/49	0/99	0/9
Width of the southern porch	8/2	0/75	0/36	1/28	2/56	0/12
Height of minaret	19	0/32	0/16	0/55	1/1	1

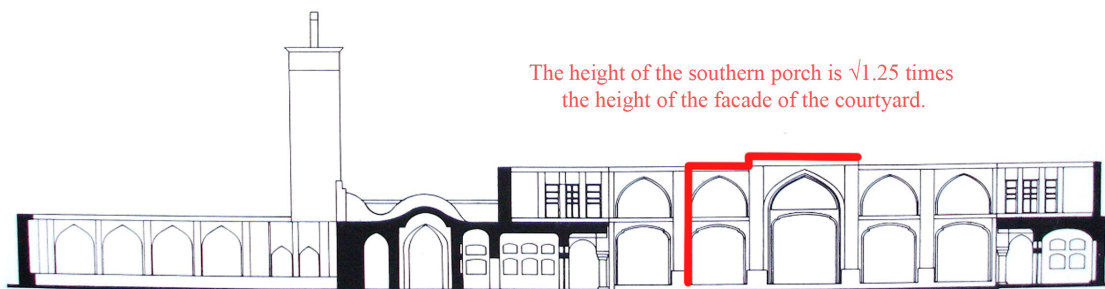


Fig. 17. The facade of the southern porch of Sepahsalar School-Mosque. Source: Haji Ghasemi, 1998, 115.

and twice the width of the porches. Finally, the height of the minaret is twice the height of the porches. As a result, the height and width of the porches, minaret, and facade of the courtyard of the mosque follow the golden and Iranian proportions (Table 8 & Fig. 18).

• **Filsouf – ol-Doleh school mosque**

The width of the northern and southern porches compared with the height of the facade of the courtyard shows golden proportions. Moreover, the proportion of the width to the height of the porches is  $\sqrt{3}$ . The height hierarchy in the mosque indicates that the height of the facade of the courtyard has a proportion of  $\sqrt{1.25}$  times the height of the northern and southern porches and a proportion of  $\sqrt{3}$  times the minaret (Fig. 19). Furthermore, the height of the porches is equal, and  $\sqrt{3}$  times the width of these porches. Finally, the height of the minaret is  $\sqrt{2}$  times the height of the northern porch and is close to the golden proportions when considering it with the southern porch. Thus, the height and width of the porches, minaret, and facade of the courtyard of the mosque follow the golden and Iranian proportions (Table 9 & Fig. 20).

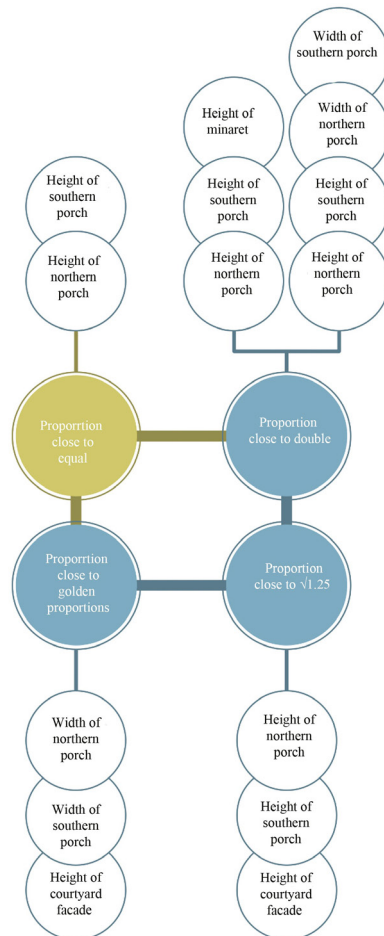


Fig. 18. Analysis of Height Proportions of Sepahsalar School Mosque of Tehran. Source: Author.

Table 8. Height proportions of Sepahsalar School-Mosque of Tehran (Unit: Metric system). Source: Author.

Sepahsalar School Mosque	Dimensions	Height of the facade of the courtyard	Height of the northern porch	Height of the southern porch	Height of the minaret
Dimensions	-	8	9/5	9/5	20
Height of the facade of the courtyard	8	1	1/19	1/19	2/5
Height of the northern porch	9/5	0/84	1	1	2/1
Height of the southern porch	9/5	0/84	1	1	2/1
Height of the minaret	20	0/4	0/47	0/47	1
Length of the yard	25/5	0/31	0/37	0/37	0/78
Width of the yard	25/5	0/31	0/37	0/37	0/78
Width of the northern and southern porches	5	1/6	1/9	1/9	4

The height of minaret is  $\sqrt{3}$  times the height of the courtyard facade.

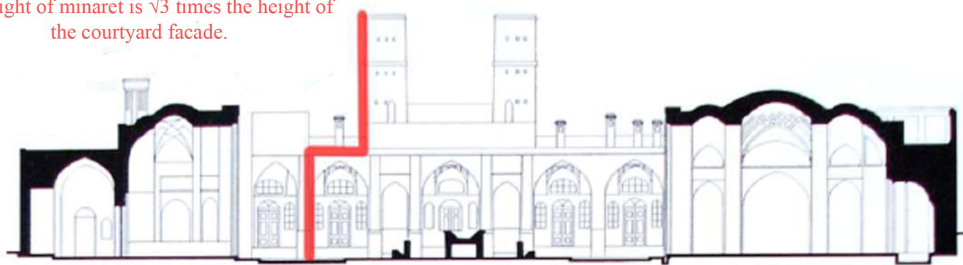


Fig. 19. The eastern facade of the courtyard in Filsouf-ol-Doleh School mosque. Source: Haji Ghasemi, 1998, 131.

Table 9. The height proportions of Filsouf ol-Doleh School mosque of Tehran. Source: Author.

Filsouf – ol-Doleh School mosque	Dimensions	Height of the courtyard facade	Height of the northern porch	Height of the southern porch	Height of the minaret
Dimensions	-	8	9/2	8/7	13/7
Height of the courtyard facade	8	1	1/15	1/09	1/71
Height of the northern porch	9/2	0/87	1	0/94	1/49
Height of the southern porch	8/7	0/92	1/06	1	1/57
Height of the minaret	13/7	0/58	0/67	0/63	1
Length of the yard	23/5	0/34	0/39	0/37	0/58
Width of the yard	18/4	0/43	50	0/47	0/74
Width of the northern and southern porches	5	1/6	1/84	1/74	2/74

• Marvi school-mosque of Tehran

The height hierarchy in this school-mosque indicates that the height of the courtyard facade has a proportion of  $\sqrt{3}$  to the height of the northern and southern porches and a proportion of  $\sqrt{2}$  to the eastern and western porches. Furthermore, the height of the northern and southern porches is equal, and the height of the western and eastern porches is also equal. However, these porches have

a ratio of  $\sqrt{1.25}$  to one another. The height and width of the northern and southern porches have golden proportions, and the height and width of the eastern and western porches indicate the proportion of  $\sqrt{2}$  to one another (Fig. 21). As a result, the height and width of the porches and courtyard facade of the mosque follow the golden and Iranian proportions (Table 10 & Fig. 22).

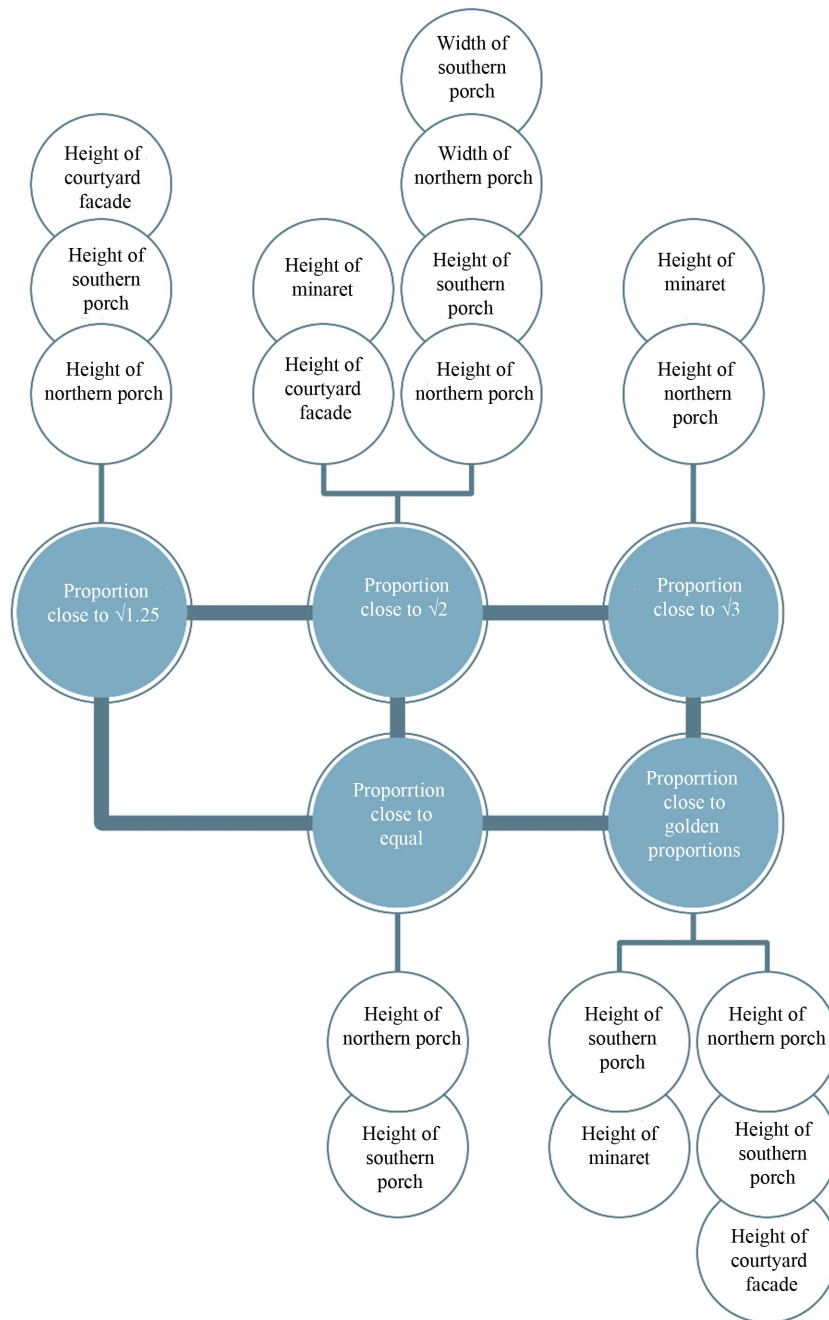


Fig. 20. Analysis of the height proportions of Filsouf-ol-Doleh School mosque of Tehran. Source: Author.

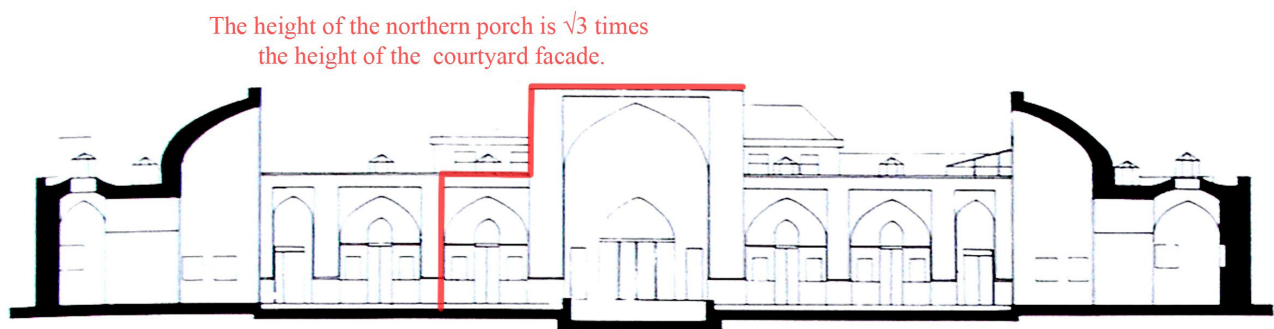


Fig. 21. The facade of the northern porch in Marvi School mosque. Source: Haji Ghasemi, 1998, 138.

Table 10. Height proportions of Marvi School mosque of Tehran (Unit: metric system). Source: Author.

Marvi school mosque of Tehran	Dimensions	Height of the courtyard facade	Height of the northern and southern porches	Height of the eastern and western porches
Dimensions	-	6	10/2	9/1
Height of the courtyard facade	6	1	1/7	1/51
Height of the northern and southern porches	10/2	0/59	1	0/89
Height of the eastern and western porches	9/1	0/66	1/12	1
Length of the yard	44	0/13	0/23	0/2
Width of the yard	30	0/2	0/34	0/3
Width of the northern and southern porches	6	1	1/7	1/52
Width of the eastern and western porches	6/5	0/92	1/57	1/4

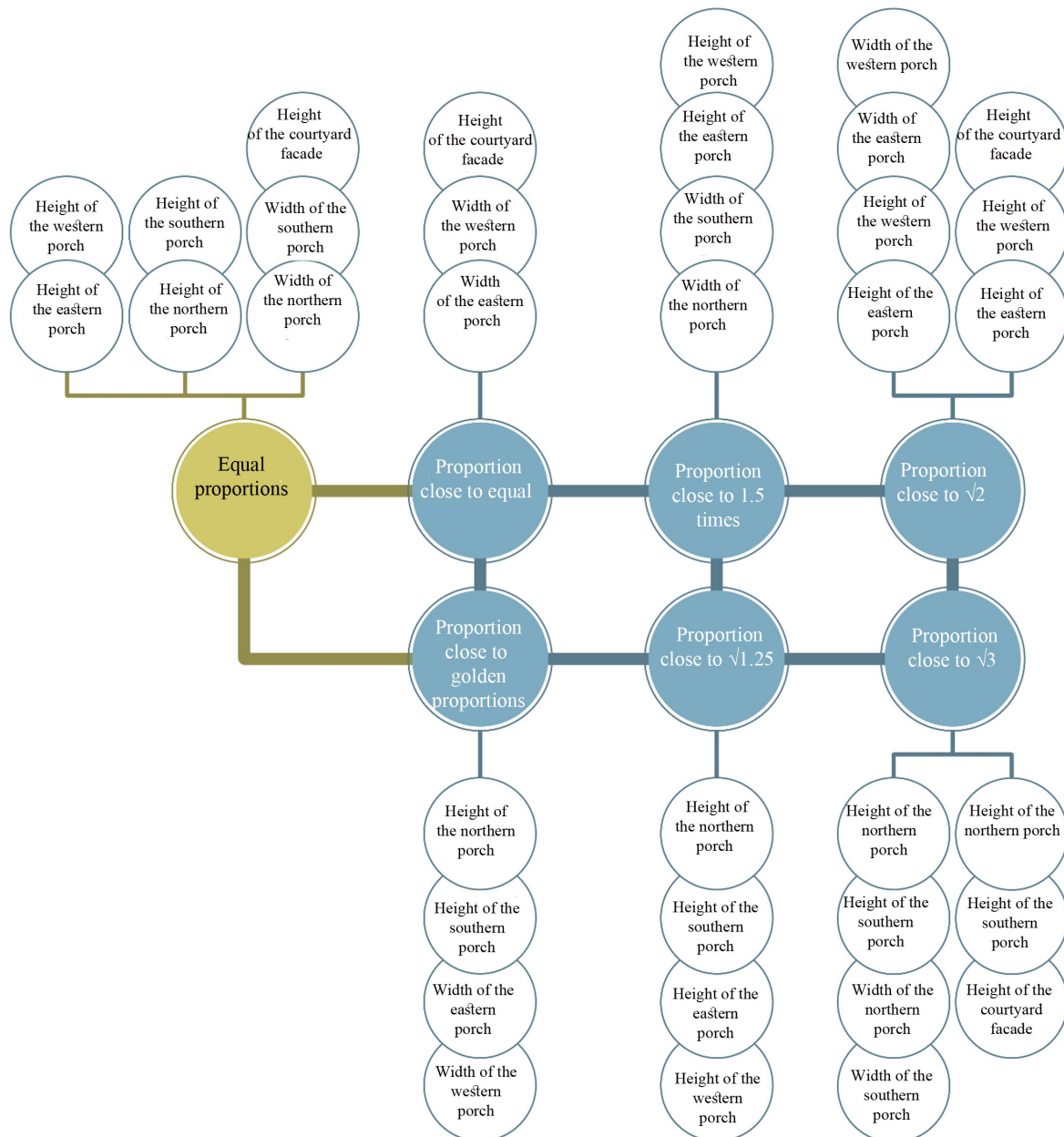


Fig. 22. Analysis of the height proportions of Marvi (Fakhriyeh) school mosque of Tehran. Source: Author.

### Data Analysis

Considering the proportion graphs and analysis of the height hierarchy, it can be concluded that all the height elements in the nine case studies, including the height of the courtyard, the shortest porch of the complex, and the highest minaret or dome, follow the golden and Iranian proportions. The beautiful and calculated adjacency of these elements has created a visually and geometrically integrated and valuable complex. Table 11 shows the results of analyzing nine case studies. Among the case

studies, the height of the six shortest porches and the facade of the courtyard had a proportion close to the golden proportions. Also, in five case studies, the highest porch had a height twice the height of the courtyard. In seven cases, the dome and minaret had a proportion of close to 1.5 times, and another one had golden proportions with the height of the southern porch. Finally, in two mosques, the height of the two floors of the courtyard had proportions close to 1.118 to one another (Table 11).

Table 11. Investigating the similar patterns and proportions in 9 case studies. Source: Author.

Row	School-mosque	The height of the shortest porch and the facade of the courtyard have proportions close to the golden proportion.	The southern porch (The highest( has a height of nearly twice the height of the courtyard facade.	The dome is nearly 1.5 times the height of the southern porch, and the minaret has the golden proportions of the southern porch.	The height of the two floors of the courtyard facade has a proportion close to 1.118 to one another.
1	Imam Mosque	*	*	*	
2	Jame Mosque				*
3	Hakim Bashi Mosque	*	*	*	
4	Sheikh Abdolhossein School Mosque		*	*	*
5	Shahid Motahari School Mosque	*	*	*	
6	Moayer Al-mamalik Mosque and Seminary	*	*	*	
7	Sepahsalar School Mosque	*		*	
8	Filsouf ol-Doleh School Mosque			*	
9	Marvi School Mosque	*			

**Conclusion**

Proportions have been used for the arrangement of the height elements and the establishment of the visual order in all the school mosques under study. A hierarchy of height proportions can be considered for each mosque, from the entrance to the wall of the courtyard and the peak of the minarets, and each element has been placed in the complex in proper proportions. Moreover, such a hierarchy avoids visual damage to the adjacent height elements and the whole complex. The proportion graph of the height elements to one another, disregarding their adjacency, indicated that in nine analyzed samples, all the height elements, from the height of the courtyard facade to the shortest porch of the complex and the highest minaret, had the golden proportions (1.681) and Iranian proportions ( $\sqrt{2}, \sqrt{3}, \sqrt{1.25}$ ). Based on the patterns in

the samples, the shortest porch of the complex had proportions close to the golden proportions of the height of the facade of the courtyard. The height hierarchy also indicated that in all the buildings, the porches, domes, or minarets had the golden proportions or Iranian proportions to the shortest porch of the complex, and the shortest porch of the complex had the proportions close to the golden proportions with the height of the facade of the courtyard. In four buildings, the southern porch (the highest) had a height of twice the height of the facade of the courtyard, and in three buildings, the height of the dome had a proportion close to 1.5 times the height of the southern porch (the highest), while this porch had a proportion close to the golden proportions to the height of the minaret of these three buildings (Fig. 23).

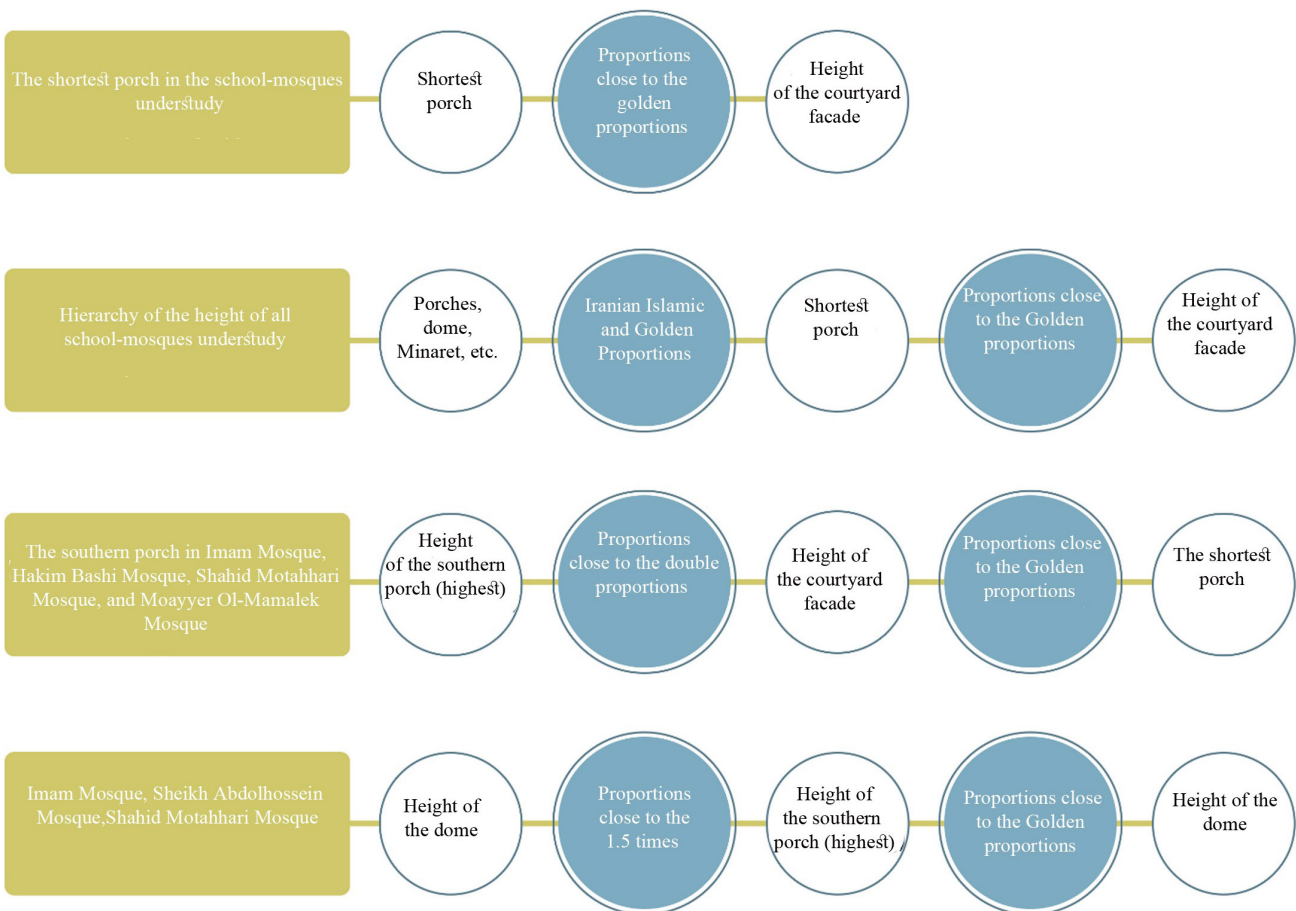


Fig. 23. The similar patterns and hierarchy. Source: Author.

## Declaration of No Conflict of Interest

It is declared that there is no conflict of interest in conducting the current research.

## References list

- Abbas, S. (2017). Golden ratio. *Resonance*, 22(1), 51-60.
- Abbasi, N., Valibeig, N. & Arya, N. (2021). A Comparative Study of the Entrance Circulation of Abbasi Jame' Mosque and Seyyed Jame' Mosque in Isfahan from the Perspective of the Persian Geometric Proportion in an Architectural Plan. *The Culture of Islamic Architecture & Urbanism*; 6(1), 39-51. doi: 10.52547/ciauj.6.1.39
- Afshar, I. (1965). *Savad va biyaz [Black and white]*. Asatir.
- Akhtaruzzaman, M. & Shafie, A. A. (2011). Geometrical Substantiation of Phi, the Golden Ratio and the Baroque of Nature, Architecture, Design and Engineering. *International Journal of Arts*, 1(1), 1-22. doi: 10.5923/j.arts.20110101.01
- Amirkhani, A., Baghai, P. & Bamanian, M. R. (2008). Investigating the transformation of the proportions governing the timchehs of Iran in the Qajar era. *Journal of Honar ha-ye Ziba*, (37), 48-39. url : [https://jhz.ut.ac.ir/article\\_27947.html](https://jhz.ut.ac.ir/article_27947.html)
- Ansari, M., Okhovat, H. & Taghavi, A. A. (2011). A research on the historical course of proportion adjustment systems in architecture with an emphasis on practical and aesthetic considerations. *Mah-e Honar*, (11), 58-46.
- Attarian, K., Momeni, K. & Masudi, Z. (2016). The study of Courtyard Proportions in Safavid Mosques of Isfahan. *Motaeate-e Tatbighi-e Honar*, 5(10) 67-82.
- Ayatollahi, H. (1984). Golden ratios in art. *Journal of Art*, (7), 108-129.
- Balilan, L. & Hasanpour Lamar, S. (2019). Geometric patterns and golden proportions, the common language of architecture and art in the historical village of Abyaneh. *Journal of Architecture in Hot and Dry climate*, (9), 68-45. Dor: 20.1001.1.26453711.1398.7.9.3.2
- Bamanian, M. R. (2001). Moghadame-ye bar naghsh va karbord-e peymon dar memari-ye Irani [An introduction to the role and application of Peymon (Module) in Iranian architecture]. *Journal of Modarres-e Honar*, (1), 1-10.
- Bamanian, M. R., Okhovat, H. & Beqaei, P. (2011). *Application of geometry and proportions in architecture*. Helle/ Tahan.
- Bicknell, M. & Hoggatt Jr, V. E. (1969). Golden triangles, rectangles, and cuboids. *The Fibonacci Quarterly*, 7(1), 73-91.
- Binabadi, N. (2013). *The application of motif geometry in Islamic architecture*. Arena.
- Buzjani, A. M. B. M. (1997). *Iranian geometry; the application of geometry in practice* (S. A. R. Jazbi, Trans.). Soroush. [in persian]
- D. K. Ching, F. (1979). *Architecture: Form, Space and Order*. John Wiley & Sons Inc.
- Dabbour, L. M. (2012). Geometric proportions: The underlying structure of design process for Islamic geometric patterns. *Frontiers of Architectural Research*, 1(4), 380-391. doi: 10.1016/j.foar.2012.08.005
- Doczi, G. (1981). *The Power of Limits: Proportional Harmonies in Nature, Art and Architecture*. Shambhala.
- Dunlap, R. A. (2008). *The golden ratio and Fibonacci numbers*. World Scientific.
- Embi, M. R. & Abdullahi, Y. (2013). Evolution of Islamic Geometrical Patterns. *Frontiers of Architectural Research*, 2(2), 243-251. doi: 10.1016/j.foar.2013.03.002
- Etemad Al-Saltaneh, M. (1876). *Marat al-buldan-e Naseri [The mirror of Naseri cities]*. Dar al-tab'a-ye dolati.
- Etemad Al-Saltaneh, M. (1976). *Al-ma'aser va al-asar [Good works and effects]*. Sanai.
- Falbo, C. (2005). The Golden Ratio: A Contrary Viewpoint. *The College Mathematics Journal*, 36(2), 123. doi: 10.2307/30044835
- Ferwati, M. A., Mandour, M. S. & Mandour, M. A. (2008). Proportions and Human Scale in Damascene Courtyard Houses. *International Journal of Architectural Research*, 2(1), 247-263. doi: 10.26687/archnet-ijar.v2i1.185
- Fletcher, R. (2006). The Golden Section. *Nexus Network Journal*, 8(1), 67-89. doi: 10.1007/s00004-006-0004-z
- Friedenberg, J. (2012). Aesthetic Judgment of Triangular Shape: Compactness and Not the Golden Ratio Determines Perceived Attractiveness. *I-Perception*, 3(3), 163-175. url: <https://www.ncbi.nlm.nih.gov/pmc/articles/PMC3485848/>
- Gangwar, G. (2017). Principles and Applications of Geometric Proportions in Architectural Design. *Journal of Civil Engineering and Environmental Technology*, 4(3), 171-176.
- Golombek, L. & Wilber, D. (1988). *The Timurid Architecture of Iran and Turan*. Princeton University Press.
- Haji Ghasemi, C. (1998). *Treasure of Iran's Islamic Architecture Culture: Tehran's Religious Buildings*. Documentation and Research Center, Faculty of Architecture and Urban Planning, Shahid Beheshti University.
- Hejazi, M. (2005). Geometry in nature and Persian architecture. *Building and Environment*, 40(10), 1413-1427. doi: 10.1016/j.buildenv.2004.11.007
- Hejazi, S. A. M. (2008). Sacred Geometry in Nature and Persian Architecture. *Journal of History of Science*, (7), 17-44. url: [https://jih.s.ut.ac.ir/article\\_22386.html](https://jih.s.ut.ac.ir/article_22386.html)
- Hosseini Balaghi, A. (1971). *Tarikh-e Tehran [History of Tehran]*. n.p.
- Iosa, M., Morone, G. & Paolucci, S. (2018). Phi in physiology, psychology and biomechanics: The golden ratio between myth and science. *Biosystems*, 165, 31-39. doi: 10.1016/j.biosystems.2018.01.001
- Kayani, M. Y. (2008). *Iranian Architecture of the Islamic Period*. SAMT.
- Khavas, M. & Bemanian M. (2022). Proportions in the height components of the four-porch mosques. *Journal of Reserches in Islamic Architecture*, 10(4), 59-81. doi: 10.52547/jria.10.4.1

- Krier, R. (1988). *Architectural Composition*. Academy Editions
- Kurt Grütter, Y. (1987). *Ästhetik der Architektur*. Kohlhammer.
- Leopold, F. (2006). Geometric Concepts in Architectural Design, Proceedings of the 12th International Conference of Geometry and Graphics.
- Lu, P. J. & Steinhardt, P. J. (2007) Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture. *Science*, 315 (5815), 1106-1110. doi: 10.1126/science.1135491
- Madden, C. (2005). *Fib and Phi in music: The golden proportion in musical form*. High Art Press.
- Mahdipour, M. & Saraj, F. M. (2012). The Application of Golden Proportion in the Façades & Ornaments of Quadruple Vaulted Porticos of Jami Mosque in Isfahan, Iran. *Journal of Civil Engineering and Urbanism*, 2(3), 97-101.
- Markowsky, G. (1992). Misconceptions about the Golden Ratio. *The College Mathematics Journal*, 23 (1), 2. doi: 10.1080/07468342.1992.11973428
- Mostafavi, M. (1982). *Asar-e tarikhi-e Tehran: Amak-e Motebareke [Historical monuments of Tehran: holy places]* (M. Mohades, Ed.). Anjoman Asar-e Meli.
- Nabavi, F. & Ahmad, Y. (2016). Is There Any Geometrical Golden Ratio in Traditional Iranian Courtyard Houses? *International Journal of Architectural Research: ArchNet-IJAR*, 10 (1), 143. doi: 10.26687/archnet-ijar.v10i1.744
- Najafkalipur Kalantari, N., Etesam, I. & Farah, H. (2011). Check in architectural geometry and proportions of the golden Iran (Case Study: Tabriz traditional houses). *Urban Management*, (46), 491-477.
- Najib Oglu, G. (1996). *Geometry and Ornament in Islamic Architecture*. Oxford University Press.
- Naseri, H., Attarian, K. & Amini Farsani, Z. (2022). Investigation the theory of proportions adjustment system in historical houses of Qajar period in Khorramabad. *Andisheh Memari publication*, 6 (14), 73-88. doi: 10.30479/at.2022.11167.1260
- Noghrekar, A. H. (2008). *Dar amadi bar haviat-e Islami dar memari [An introduction to Islamic identity in architecture]*. Ministry of Housing and Urban Development. Office of architecture and urban design: Payam Sima design and publishing company.
- Noghrekar, A. H. (2010). *Mabani nazari-ye memari [Theoretical Foundations of Architecture]*. Payam Noor University.
- Pimia, M. K. (2004). *Sabkshenasi-e Memari-e Irani [Iranian architectural stylistics]*. Soroush-e Danesh.
- Pope, A. (1976). *Persian Architecture*. Soroush Press.
- -Posamentier, A. S. & Lehmann, I. (2012). *The glorious golden ratio*. Prometheus Books.
- Pourmand, H., Yarali, Z., Afhami, R. & Abbasi, P. (2014). A Study on Proportions of Entrance Façade of Sheikh Lotfolah, Ja'me Abbasi, Hakim Mosques and Chahar-Bagh School in Isfahan. *Journal of Architecture and Urban Planning*, 6 (12), 153-163. doi: 10.30480/aup.2014.107
- Putz, J. F. (1995). The Golden Section and the Piano Sonatas of Mozart. *Mathematics Magazine*, 68 (4), 275. doi: 10.2307/2690572
- Saatian, R. & Moghaddasi, T. (2010). Morori bar barkhi masajed-e shakhes-e shahr-e Tehran [An overview of some prominent mosques in Tehran]. *Ghardeshgari*, 12(26), 30-33.
- Shamaileh, A. A. (2022). Study and investigate perceptions of Jordanian college students on the golden ratio proportions. *Computers and Electrical Engineering*, (103). doi: 10.1016/j.compeleceng.2022.108317
- Sharp, J. (2002). Spirals and the Golden Section. *Nexus Network Journal*, 4(1), 59-82.
- Tayefeh, E. (2016). *Idea & creativity in persian architecture*. (3rd ed.). Elm-e Memar.
- Ziaieniya, M. H. & Hashemi Zarj abad, H. (2016). The Golden Proportion and System of Islamic-Iranian proportions in Qaen Mosque *Journal of Maremat va Memari-e Iran*, (11), 89-99. URL: <http://mmi.aui.ac.ir/article-1-259-fa.html>
- Zoka, Y. (1967). *Tarikhche va rahna-ye kakhha va sakhtemanha-ye arg-e saltanati [History and guide of the palaces and buildings of Royal Citadel]*. Vezarat Farhang va Ershad Islami.

**COPYRIGHTS**

Copyright for this article is retained by the author(s), with publication rights granted to the Bagh-e Nazar Journal. This is an open-access article distributed under the terms and conditions of the Creative Commons Attribution License (<https://creativecommons.org/licenses/by/4.0/>).

**HOW TO CITE THIS ARTICLE**

Hayaty, H. (2024). Investigating the Geometrical Proportions in the Height Elements and Components of Qajar Era Mosque-Schools in Tehran. *Bagh-e Nazar*, 21(132), 61-80.

DOI: 10.22034/BAGH.2024.405521.5415

URL: [https://www.bagh-sj.com/article\\_194859.html?lang=en](https://www.bagh-sj.com/article_194859.html?lang=en)

