Bagh-e Nazar

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Original Research Article

Analysis of Fractal Geometry in the Maqeli Tiled Spandrel of the Northern Iwan in Hakim Mosque in Isfahan*

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Abstract

Problem statement: Geometric knots are prevalent patterns in Islamic architectural designs characterized by complexity, Plurality, rhythm, and balance in drawing, extending beyond Euclidean geometry. Fractal geometry as a foundation of forming the universe and nature is non-Euclidean. Many artists, besides knowing geometry, use nature as a model, thus creating their artwork perhaps unconsciously or consciously based on fractal geometry. One of these enduring manifestations is the Hakim Mosque in Isfahan. Since behind these beautiful works, there are hidden rules, and knowing these rules elevates our understanding to achieve beauty, the study of fractal geometry in these works reveals their hidden angles and beauty. This research seeks to answer the question: How can fractal geometry be used to analyze the geometric structure of the Maqeli tiled spandrel of the northern Iwan in Hakim Mosque?

Research objective: This research attempts to analyze the fractal geometry system in the Maqeli tiled spandrel of the northern Iwan in Hakim Mosque.

Research method: This research is applied in nature, utilizing field data collection, library and internet sources. The methodology is descriptive and analytical. It is noteworthy that the analysis of the knot dimensions was conducted using software.

Conclusion: By analyzing the Maqeli patterns from a fractal perspective (self-similarity and dimension), the result of the research was the dimensions of these geometric patterns were fractional. These dimensions, calculated using two methods (logarithmic box-counting and software), showed minor differences and completely matched fractal dimensions. Furthermore, by linearizing the patterns with AutoCAD software and examining the hidden geometry, self-similarity, and repetition in the secret geometry of the designs as well as in the geometric patterns of the Maqeli tiles were revealed.

Keywords: *Fractal geometry, Geometric Patterns, Fractional dimension, Self-similarity, Maqeli Tile, Hakim Mosque.*

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Introduction

In most cases, nature, with its mathematical patterns, has inspired artists and engineers. Spiritual architecture, as a bio-mimetic approach, interprets sacred natural architecture. Islamic architecture employs symbolic abstract geometry derived from nature (Abdelsalam & Ibrahim, 2018, 27). Since fractal geometry is the foundation of nature's creation, and artists have utilized formal and contextual patterns from nature for architectural decorations, one can observe this pattern adoption in Islamic architecture by examining the hidden and apparent geometries in the decorations. Therefore, it is possible to trace elements of fractal geometry in these works. Geometric patterns contain significant aspects that warrant attention. Consequently, designers have conducted numerous studies analyzing symmetry and some periodic or quasi-periodic patterns to uncover the secrets of their construction (Khamjane & Benslimane, 2017, 5). Beyond the geometric motifs of each artwork lie mathematical relationships. For example, a small shape that repeatedly appears in a pattern, creating a design, must maintain its proportion and shape, reflecting the artist's taste. This pattern is abundantly observed in the tilework of the Safavid period. The Hakim Mosque, a masterpiece from the Safavid era, is adorned with Maqeli tiles, showcasing remarkable beauty and the skill of our artists, revealing the order and hidden dimensions of their thoughts. A detailed study of the fractal geometry in these works reveals their hidden angles, charm, and beauty. Furthermore, understanding the fractal structures in the tilework of this building can aid in the precise restoration and creation of new architectural decorations. Thus, this research aims to study fractal geometry in some of the Maqeli tilework of the northern Iwan of Hakim Mosque, examining pattern repetition, finding the rhythm of the motifs, and the similarities that fractal fragmentation has with the multiplication in Islamic geometric patterns, and analyzing them using fractal geometry principles (self-similarity and fractional dimension). This study seeks to determine how fractal geometry can be used to analyze the geometric structure of the Maqeli tiles in the spandrel of the northern Iwan of Hakim Mosque in Isfahan.

Literature Review

Fractal geometry applies to visual arts such as architecture, urban planning, painting, and sculpture (Mobini & Fatholahi, 2015). Common features exist between the geometric structures of nodes and fractal geometry (Balilan Asl et al., 2011; Hayati & Aghamohammadi,2016;Beikzadeh&Forouzanfar,2015). Fractal geometry shares similarities with Islamic patterns, such as unit cell repetition and possessing the law of iteration (Miriyan, 2018). Researchers have explored fractal shapes in the decorations of traditional architectural structures like the Al-Sultan Hasan School (Ismail Ismail Attia, 2020; Abdelsalam & Ibrahim, 2018). Others have uncovered some aesthetic aspects of mathematical properties, such as golden ratios and fractal geometry, in Islamic art geometry at the Atarin School (Khamjane & Benslimane, 2017). Decorations of the Sheikh Lotfollah School in Isfahan, such as muqarnas under the dome, plans, sections, and architectural facades, possess fractal geometry characteristics (Rezazade, 2020). Artists have examined fractal geometry in Islamic patterns and created fractal artworks using Islamic motifs by repeating patterns at various scales and filling empty spaces with the same motifs or arrangements (Lin & Kaplan, 2023; Webster, 2013). architectural Kufic script and Maqeli patterns in Hakim Mosque have been studied structurally and geometrical aspects (Zomrashidi, 2002; Hazrat Gholizadeh, 2015; Keshavarzi Myandashti & Faizabi, 2017; Qaragozlu, 2017; Oulad Qobad, 2017). The geometry of the tile and brickwork patterns behind the vault of the Hakim Mosque has been analyzed in terms of visual techniques, focusing on balance, order, and the Gestalt approach (Sartipi & Valibeyg, 2018b).

Based on conducted research, studies on fractal geometry in Safavid-era tile patterns are scarce, and it is noted that in these studies, the calculation of fractal dimension has not been accurately performed with software and has focused on self-similarity properties. Moreover, no research was found regarding fractal geometry in the Maqeli tilework of the Hakim Mosque in Isfahan. Therefore, this study examines the precise characteristics and features of fractal geometry (self-similarity and dimension) in the Maqeli tiled spandrel of northern Iwan in Hakim Mosque in Isfahan.

Theoretical Foundations

• Fractal geometry

Fractals are referred to as complex structures whose most important characteristic is self-similarity. The word "fractal" is derived from the Latin root "fractus," meaning irregularly broken, implying that if you break a part of a fractal into smaller pieces, the whole is preserved (Frantz & Crannell, 2011,140). The term "fractal" was coined by Benoit Mandelbrot[1] in the 1960s and 1970s. Following the introduction of fractal geometry by Mandelbrot, irregularities, which are simultaneously regular, were analyzed using fractal properties.

Fractal geometry is a new language with which one can converse, It, allows you to describe cloud formations as accurately as an architect can describe a building (Barnsley, 2012, 1). Fractals are shapes that, unlike Euclidean geometric shapes, do not conform to Euclidean regularity, and their irregularity is consistent at all scales.

• Characteristics of fractals

- Self-similarity

Fractal geometry is primarily based on the idea of self-similarity. Self-similarity means that the parts of a shape behave similarly at different scales (Blanco et al., 2020, 171). To get a self-similar, a shape must be divided into parts where smaller copies resemble the whole shape (Aswathy, 2016, 102). A fractal form is recognized as a fragmented geometric shape that can be subdivided into an infinite number of reduced-size copies of the whole form. Self-similarity is one of the prominent features of fractal forms (Ismail Ismail Attia, 2020, 1). Fractals represent an infinite concept and nested worlds. They emerge through feedback mechanisms recognized by scientists as feedback loops. The processes that generate fractals are simple feedback loops that repeat actions (Peitgen & Richter, 1986). If you look at the shore of a continent, you will see a rugged shape, and if you zoom in on a small part of the shore, you will still see a similar ruggedness. However, this similarity is approximate, and the significant feature is the scale invariance of nature, which makes fractal geometry helpful in modeling various phenomena (Fig. 1).

scalable shape means that similar patterns exist at different scales within the specified range. Magnification presents a small part of a shape that appears identical to the whole image, and reducing the entire image produces something similar to a small portion (Eglash, 1999,17).

- Fractional dimension

Fractal geometry deals with studying intricate geometric problems, and the concept of dimension is a measure of the complexity of this intricacy. In other words, dimension in fractal geometry is a quantitative measure of the level of complexity in the representation of a pattern (McClure et al., 2022,42). Typically, dimension is considered as an integer. For example, in Euclidean geometry, we believe the dimension of a point, is zero, the dimension of a line, is one, and the dimension of a plane, is two. However, fractal phenomena have fractional dimensions (Fig. 2). As observed in the figure, the exponent D represents the dimension, which is an integer in Euclidean shapes like squares and cubes. but fractional in fractal shapes. Fractal shapes exhibit characteristics such as texture, complexity, fracturing, and roughness. The higher these parameters are, the greater the fractal dimension (Nayak & Mishra, 2017, 33) (Table 1).

- Similarity dimension

The most superficial fractal dimension associated with fractals generated from iterated function systems is the similarity dimension (Glass, 2011,25). In all self-similar structures, there is a relationship between the scaling factor and the number of subdivided smaller parts upon which the original structure is based.

If N is the number of parts and S is the number of

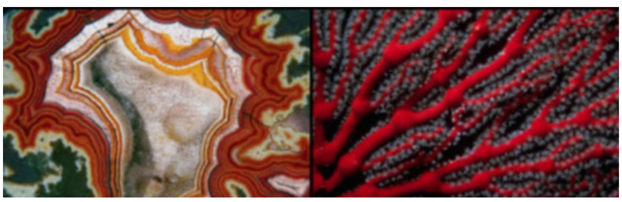


Fig.1. Self-similarity in nature. Source: Jokar, 2005.

parts that the initial line is divided into (scale), the fractal dimension is given by Falconer (2014):

Formula 1. $$D=\frac{\text{Log N}}{\text{Log S}}$$

By using Formula 1, the similarity dimension has been calculated for several well-known fractal structures (Fig. 3). In the cantor set (Fig. 3-a), the similarity dimension is equal to D= Log 2/ Log 3=0.63. In the Sierpinski triangle (Fig. 3-b), the dimension equal to D= Log 3/ Log 2=1.58. The Koch snowflake or the Von Koch curve (Fig. 3-c) has a similarity dimension of D= Log 4/ Log 3=1.26.

Unfortunately, the similarity dimension is meaningful only for a small class of self-similar sets, so it has limited practical applications (Delkhosh, 2017, 7).

- **The box-counting method**

the box-counting method is one of the most valuable techniques for determining fractal dimensions. This method has been defined since at least the 1930s and is sometimes referred to as Kolmogorov entropy, entropy dimension, capacity dimension, metric dimension, information dimension, and logarithmic density dimension (Falconer, 2014, 41).

In this method, a grid of square-shaped boxes is placed over the image, and the size of the grid is denoted as (s). The number of boxes that cover the image is represented as N(s). The number of covering squares indicates the extent of positive and negative space within a unit area. Negative space is considered empty, while positive space forms the main shape, and the squares that do not cover the shape create gaps in the surface, leading to a dimensionality between one and two. This process is repeated by decreasing the size of boxes and counting the number of boxes covering the image N(s) (Bovill, 1996, 41-42) (Fig. 4).

The fractal dimension is calculated as Formula 2.

Formula 2. $$D_b=\frac{[\text{Log}(N(S2))-\text{Log}(N(S1))]}{[\text{Log}(\frac{1}{S2})-\text{Log}(\frac{1}{S1})]}$$

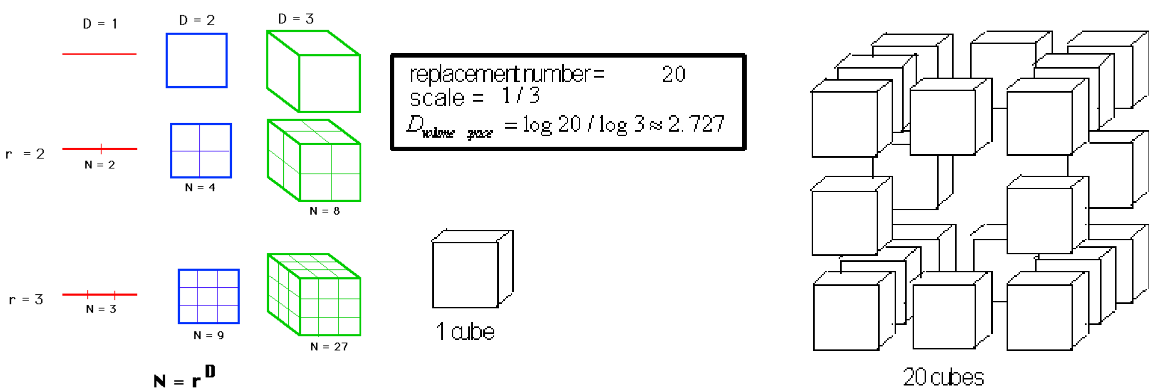
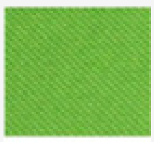
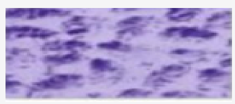
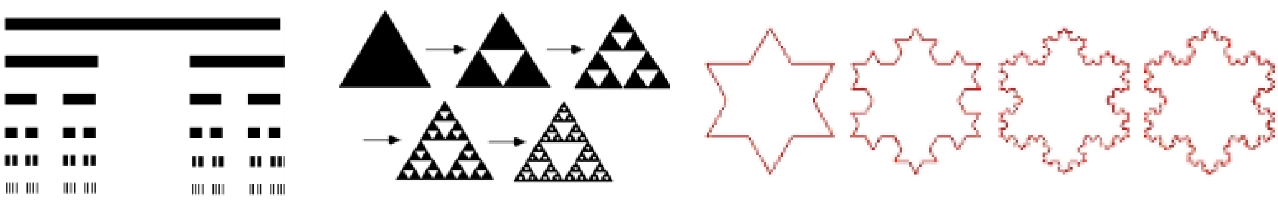


Fig. 2. A comparison of integer dimensions with fractional dimensions. Source: Right: https://www.wahl.org, Left: https://www.vanderbilt.edu.

Table 1. A comparison of the fractal dimensions of images based on texture, complexity, and roughness. Source: Nayak & Mishra, 2017.

Title	Rough Image So	Texture Image	Smooth Image	Rough Image
Image				
Fractal dimension	D=3.33	D=2.81	D=2	D=2.92



b) Sierpinski Triangle

c) Koch Snowflake

Fig. 3. Some examples of famous fractal structure. Source: a & c) Jokar, 2005, b) https://math.bu.edu.

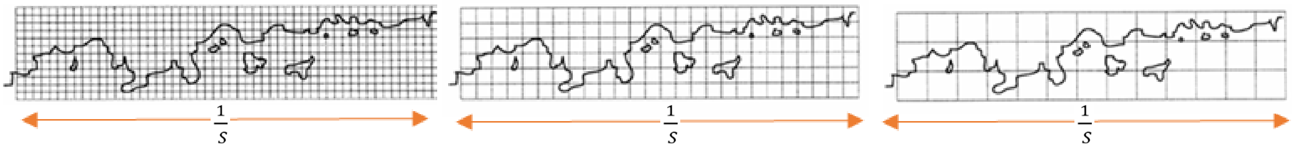


Fig. 4. Box counting grids. Source: Bovill, 1996.

• Geometric patterns

Geometric patterns are those with defined angles created by connecting curved, straight, and oblique line segments in a mathematical composition. In this type of composition, all spaces are filled using a geometric structure, and the background and patterns are essentially unified, resulting in a space saturated with geometric patterns that can even be analyzed as a texture. Experts refer to geometric patterns as "knotwork," which includes various woven geometric shapes (Navayi, 1996, 272). A knot is a collection of geometric shapes used harmoniously and complementarily against a defined background (Samaniyan, 2008, 7). Each knot has a background, or frame, and a knot unit. The working unit in knot weaving and knot making is called the "knot unit" (Amir Ghiyasvand, 2003; Saba, 2004; Bogvaran, 1983). "Alat" essentially comprises the boundary lines around the "loqat" (segments). "Loqat" refers to the shapes in the knot formed from straight lines based on regular rules and confined within a frame or background. "Loqat" can be made of wood, glass, or other material (Dehshati et al. 2019).

• Maqeli tiles

"Maqel," pronounced like "Mahfel," means a tall tower, a strong fortress, a sturdy building, or a sky-reaching minaret, and it is an Arabic word. In some instances, "Maqel" also refers to the "knotting of a camel's knee." The similarity between the knotting of a camel's knee and the intricate patterns formed by "chessboard squares" and "rectangles" has led to the naming of this tile pattern in tile work (Zomrashidi, 2002, 43). Among tile artisans, this term may be due to the complexity and intricacy required to execute these designs, necessitating significant thought and precision (Maher al-Naqsh, 2004, 8). The art of Maqeli has roots in pre-Islamic works, as evidenced by the discovery of tile patterns from the Achaemenid period, which feature tile pieces measuring 15x[7] cm with a depth of 6 to 7 cm (Zomrashidi, 2004, 15). Maqeli, or architectural tiling, is created by drawing geometric shapes such as squares, diamonds, rectangles, and intersecting parallel lines (ibid., 1994, 4).

• Mosque Hakim

- History of Hakim Mosque

Hakim Mosque (Fig. 5) is one of the four congregational mosques in Isfahan dating back to the late Safavid period in the Bab Al-Dasht area of Isfahan. It was constructed by Hakim Mohammad Davood, doctor of Shah Abbas II, between the years 1656 and 1662, on the ruins of the Jorjir Congregational Mosque (4th century AH) (Maher al-Naqsh, 1997, 16) (Fig.5).

He fled to India, and due to his company with Aurangzeb[2] and the tasks he performed for him, he became known as "Tagarob-Khan." He sent a great deal of money to his family and ordered the construction of this Mosque (ibid., 11).

a) Diyalameh portal remaining from Jorjir Mosque **b) The western porch and the northern door of Hakim Mosque**

Fig. 5. Hakim Mosque of Isfahan. Photo: Authors's archive.

- **The architecture of Hakim Mosque**

This mosque spans approximately 8000 (m^2) and is characterized as a four-iwan mosque typical of Safavid-era structures, with facades constructed from brick. The building has three entrances: northern, eastern, and western portals and the Jorjir portal (Diyalameh) (ibid.). The till work type of this mosque is Maqeli, and inscriptions include interlaced and oblique Kufic script, one, two, and three-row Kufic script (Meysami, 2009, 25). The architect of this building is Ali Beik-Isfahani, and the tiler of this mosque is Mirza Mohammad Kashipaz. The inscriptions, primarily in Thuluth script, were authored by Mohammad Reza Emami Isfahani, and the mihrab inscription of the western Shabestan was scribed by Mohammad Baqer Shirazi (Honarfar, 1971, 618). The mosque is richly adorned with ornamental motifs and inscriptions, with one of the most elaborate examples found on the spandrel of the northern Iwan (Fig. 6), subject to fractal geometry analysis.

Research Method

This research is applied in nature. The methodology is descriptive and analytical. Field data collection included direct observation and photography to complement theoretical foundations and introduce fractal structures and Maqeli patterns; Documentary sources (library) and internet resources have been utilized. Initially, the structure and foundations of fractal geometry and the tilework patterns were described. Subsequently, the analysis of these patterns based on fractal geometry was performed by linearizing them using AutoCAD software and adapting them with examples of fractal geometry. The obtained data is analyzed both quantitatively and qualitatively. After drawing the patterns and knots, their self-similarity is examined, followed by calculating their dimension using the logarithmic formula (box-counting method) and software.

Fig. 6. Northern Iwan of Hakim Mosque. Photo: Authors's archive.

The Selected Sample for Fractal Analysis

The Spandrel of the northern Iwan in Hakim Mosque, depicted precisely in Fig. 7, is characterized by a "dopanji tabldar" knot (Fig. 8). The interpretation of the image relies on the contrast between the figure and background. The background in the Spandrel of the northern Iwan can be considered as brick, with the knot pattern as the figure. Fractal analysis was conducted on these patterns (Sartipi & Valibeyg, 2018a, 2018b). In tile and brickwork, patterns and nodes are based on these elements. As mentioned, fractal shapes, despite their irregularity, exhibit order, which itself contributes to equilibrium. Thus, fractal shapes maintain balance in movement, density, and bumpiness (for example, Sierpinski triangle or contour dust) (Fig. 3). In reality, placing elements in fractals in various sizes and their repetition creates order and balance. These features are observed in the patterns on the Spandrel of the northern Iwan in Hakim Mosque as well. Visual elements of varying sizes on both sides of the frame (key motifs) and rounded elements (stars) contribute to balance in the design. Additionally, similar shapes repeated in different directions create order, rhythm, and balance (ibid.) (Fig. 9-b). The hierarchy in the visible and invisible geometry of patterns also indicates order and self-similarity (Figs. 8 & 9).

- **Self-similarity features**

First, the patterns are linearized using AutoCAD software and then analyzed to find self-similarity features. Because the arch pattern is formed from a repetitive "dopanji tabldar" knot, self-similarity in this node is also examined. Additionally, since the left and right patterns are symmetrical, fractal geometry analysis suffices for one of them.

- Self-similarity features based on star

Based on one of the stars present in the pattern, stars of various dimensions similar to that star can be found, which repeat. This repetition is observed both in the spandrel pattern and in "dopanji tabldar" knot on which the pattern is based, and this action can be repeated for each of the stars present in the pattern (Fig. 9.a).

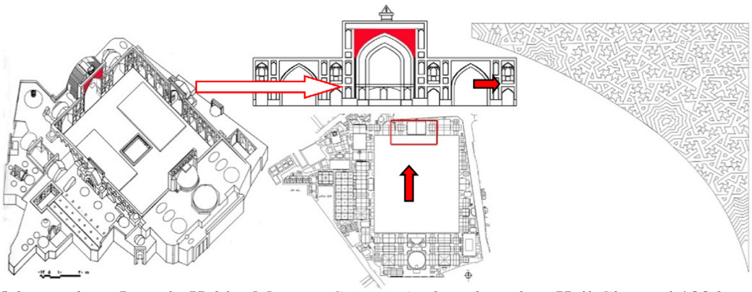


Fig. 7. The location spandrel of the northern Iwan in Hakim Mosque. Source: Authors based on Haji Ghasemi,1996.

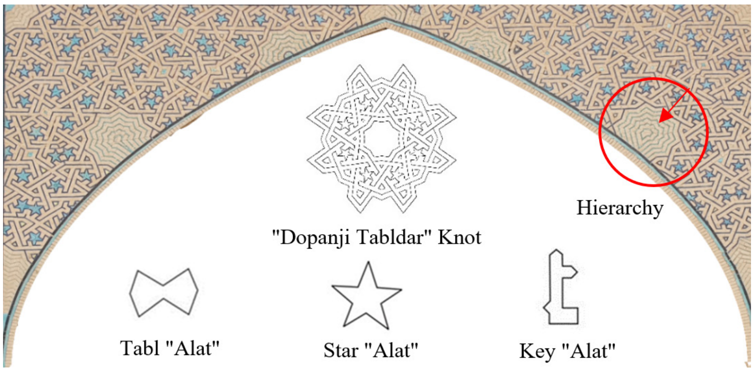
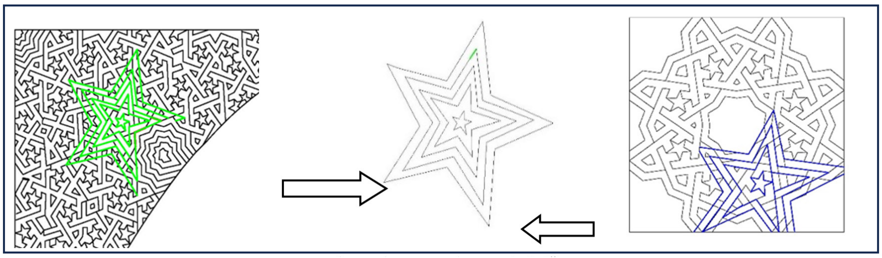
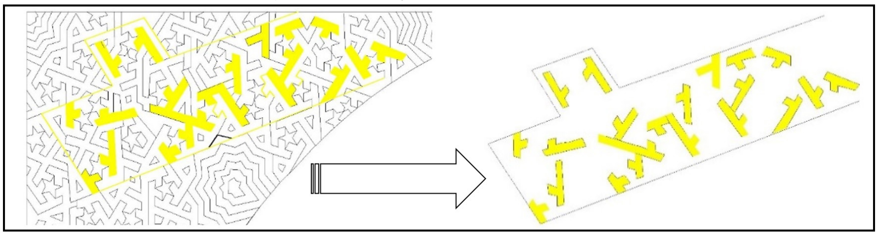


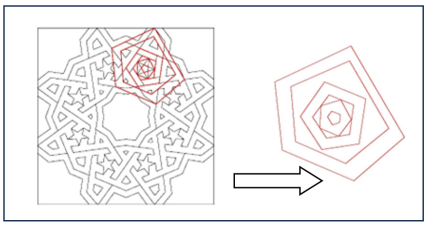
Fig. 8. The spandrel of the northern Iwan in Hakim Mosque. Source: Authors.



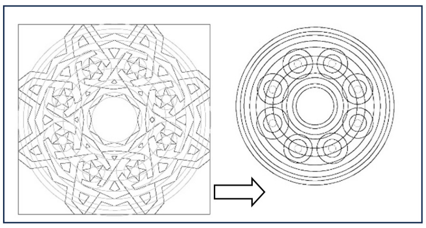
a) Self-similarity Features Based on Star.



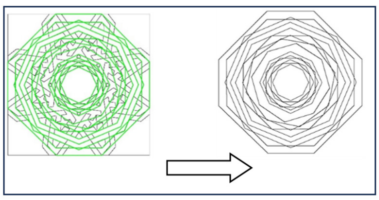
b) Self-similarity Features Based on Key “Alat”.



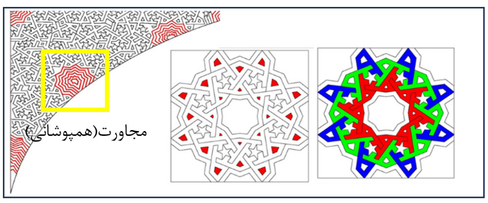
c) Self-similarity Features Based on Circle.



d) Self-similarity Features Based on the Pentagon.



e) Self-similarity Features Based on octagon.



f) Self-similarity Features in Various Parts of the Pattern.

Fig. 9. Examining self-similarity in pattern. Source: Authors.

- Self-similarity features based on key "Alat"

The extension of some lines in the tile patterns creates a key role that encompasses different sizes of keys,"Alat", which indicates a self-similarity feature (Fig. 9.b).

- Self-similarity features based on circle

The surrounding circles and circumferences around the polygons present in the pattern, and the circles surrounding the stars in various sizes, create concentric circles and circles of different sizes within them, indicating a feature of self-consistency in hidden geometry of the design (Fig. 9-d)

- Self-similarity features based on the pentagon:

If a pentagonal shape surrounds and encompasses the five-pointed star present in the pattern, similar pentagons can be drawn in various dimensions corresponding to the lines present in the pattern. This action can be repeated for all the stars present (Fig. 9-c).

- Self-similarity features based on octagon

An octagonal shape surrounding and encompassing the eight-pointed star, and also octagonal shapes corresponding to the lines of the pattern, creates concentric octagons that exhibit self-similarity features, indicative of fractal patterns in hidden geometry of the design (Fig. 9-f).

- Self-similarity features in various parts of the pattern

In different parts of the design and the nodes, the repetition of patterns in various sizes is observed, shown by different colors. The existing patterns and designs in the decorations, while maintaining their identity, also overlap and adjoin each other. The eye tends to group similar things even if they are slightly different (Fig. 9-e). This is an example of self-similarity. The repetition of these patterns can be seen in the spandrels. In these patterns, the law of continuity is also observed, where visual elements (key motifs) are repeated consecutively, and the eye tends to move along the design. Additionally, the patterns follow the principle of interpenetration, where smaller patterns tend to be placed within larger ones (ibid.) (Fig. 9). These features, which indicate repeatability and small-scale structure, are also observed in fractal shapes and indicate self-similarity.

- **Dimension calculation**

- Dimension calculation using the Box-Counting Method by logarithmic formula

To calculate the dimension of patterns, a grid of squares is placed over the desired knot, and then the number of squares covering the knot is counted. In the next step, the size of the squares is reduced, and this process continues. The size of the largest grid unit is 40 centimeters, the following grid unit size is 20 and 10, and the smallest size is 5 centimeters (Fig.10) (Table 2). calculating the box-counting dimension is lengthy, and the grid unit size must approach zero, which cannot be accurately done manually and requires software for higher precision. This will also be addressed. The dimension is calculated by using Formula 2.

$$D(40\text{-}20)=\frac{[\mathrm{Log}(26)-\mathrm{Log}(9)]}{[\mathrm{Log}(10)-\mathrm{Log}(5)]}=\frac{1.41-0.95}{1-0.69}=1.48$$

$$D(20\text{-}10)=\frac{[\mathrm{Log}(86)-\mathrm{Log}(26)]}{[\mathrm{Log}(20)-\mathrm{Log}(10)]}=\frac{1.93-1.41}{1.30-1}=1.73$$

$$D(10\text{-}5)=\frac{[\mathrm{Log}(318)-\mathrm{Log}(86)]}{[\mathrm{Log}(40)-\mathrm{Log}(20)]}=\frac{2.50-1.93}{1.60-1.30}=1.9$$

ar" knot, the basis for the arch patterns, is calculated. As before, the size of the largest grid unit is 40 centimeters, the following

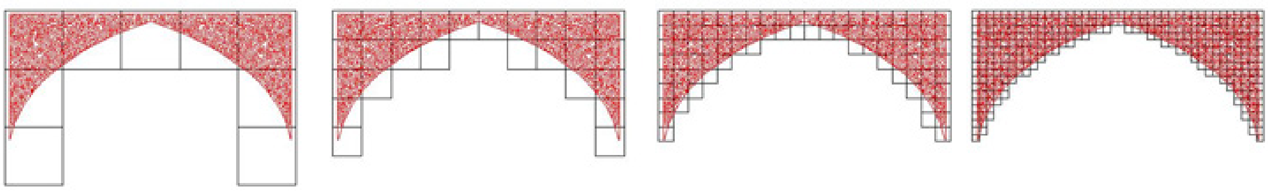


Fig.10. the grid of squares is placed over the image. Source: Authors.

Table 2. Analysis of box counting. Source: Authors

Step	Size of boxes	Number of boxes(N)	Grid size(1/s)	LogN(s)	Log (1/s)	Log(s)
1	40	9	5	0.95	0.69	1.60
2	20	26	10	1.41	1	1.30
3	10	86	20	1.93	1.30	1
4	5	318	40	2.50	1.60	0.69

grid unit size is 20 and 10, and the smallest size is 5 centimeters (Fig. 11) (Table 3).

$$D(40\text{-}20)=\frac{[\log(76)-\log(21)]}{[\log(10)-\log(5)]}=\frac{1.88-1.32}{1-0.69}=1.80$$

$$D(20\text{-}10)=\frac{[\log(244)-\log(76)]}{[\log(20)-\log(10)]}=\frac{2.38-1.88}{1.30-1}=1.66$$

$$D(10\text{-}5)=\frac{[\log(707)-\log(244)]}{[\log(40)-\log(20)]}=\frac{2.84-2.38}{1.60-1.30}=1.53$$

e Box-Counting Method with software

The dimension of both node samples was calculated using software. The software used the box-counting method, considering ten different grid sizes for the calculation. A few selected samples are presented in Table 4. For the arch knot (Fig. 10), similarly, the software calculated the dimension as D=1.69.

Discussion

In the past, architects were inspired by nature to create the tile patterns of traditional buildings. Therefore, one can find traces of fractal geometry (the geometry of nature) in these patterns. To examine fractal principles in architectural decorations, a section of the Maqeli tilework from Hakim Mosque in Isfahan was selected. Since the chosen pattern is formed by repeating the "dopanji tabldar" knot, the fractal geometry features of this knot were also examined separately. In the chosen sample, two important fractal characteristics (self-similarity and dimension) were analyzed. The self-similarity feature (including the repetition of patterns at different scales) was manifested in the patterns and their hidden geometry due to adjacency, continuity, overlap, and interpenetration (Table 5). Additionally, these patterns, like fractal shapes, exhibit order within apparent disorder (heterogeneous elements in the pattern are also orderly) and balance. The fractal dimension was calculated using the logarithmic (box-counting) method and software. The dimensions perfectly corresponded to the fractal dimension, being fractional. Based on the estimated numbers, as shown in Tables 1 & 2, the plotted graph shows log(1/s) on the x-axis and log N(s) on the y-axis (Fig. 12). As observed, the line equations show that the slopes of the lines are almost identical, indicating the consistency and coherence of the knots.

Finally, this study identified important features of fractal geometry in tile patterns. Since most patterns and nodes

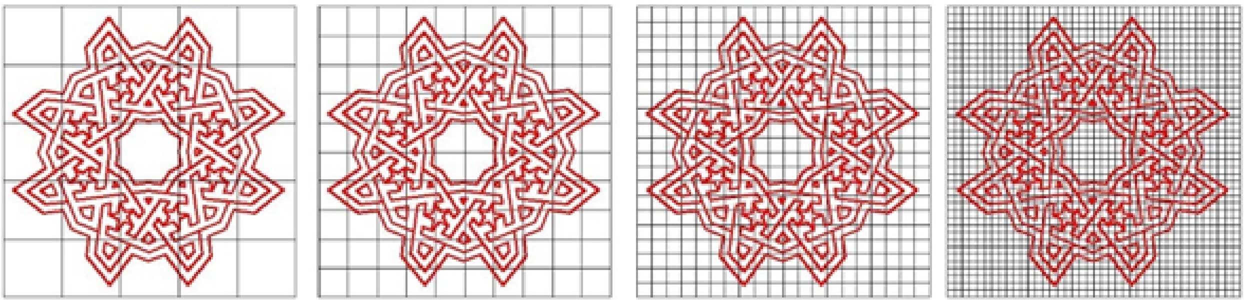
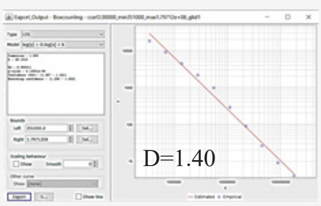
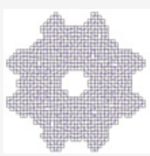
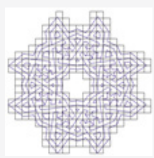
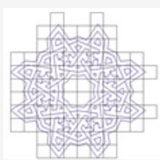
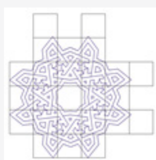


Fig.11. the grid of squares is placed over the image. Source: Authors.

Table 3. Analysis of box counting. Source: Authors.

Step	Size of boxes	Number of boxes(N)	Grid size(1/s)	LogN(s)	Log (1/s)	Log(s)
1	40	21	5	1.32	0.69	1.60
2	20	76	10	1.88	1	1.30
3	10	244	20	2.38	1.30	1
4	5	707	40	2.84	1.60	0.69

Table 4. Analysis of box counting by software. Source: Authors.

Fractal dimension calculation	Step4	Step3	Step2	Step1
				

in decorations exhibit similar characteristics to those mentioned, and because the nodes have fractional dimensions due to empty spaces and breaks, this research can explore fractal geometry features in other architectural decorations.

Previous studies have also sought to identify fractal geometry features in geometric patterns and knots. However, research on fractal geometry in Safavid-era tile patterns has been sparse, and no research was explicitly found on Maqeli tilework in Hakim Mosque in Isfahan or other buildings with Maqeli tiles. Additionally, this study calculated the fractal dimension using two methods, which were not observed in previous research.

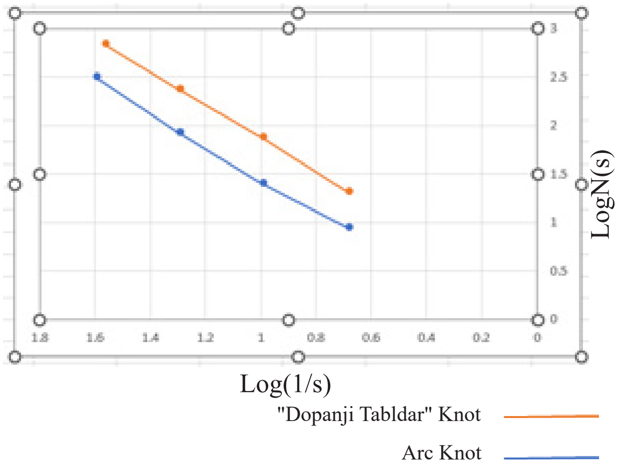


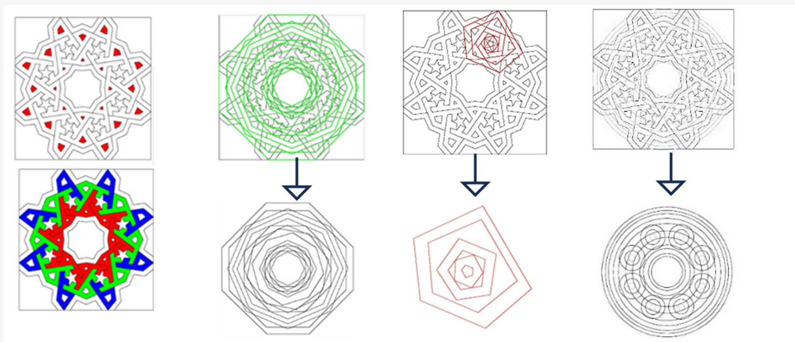
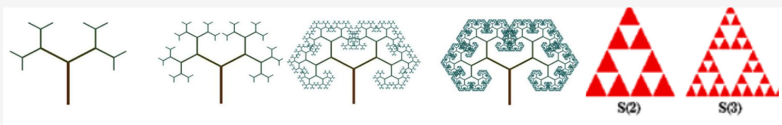
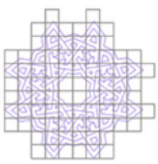
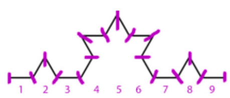
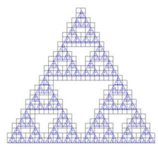
Fig. 12. Comparing the slope of graph nodes. Source: Authors.

Conclusion

This study described, analyzed, and examined the Maqeli tilework of the spandrels in the northern iwan of Hakim Mosque based on fractal geometry features. It concluded that the geometric patterns of the tiles exhibit fractal characteristics, including self-similarity. The dimension was then calculated using the (box-counting method) logarithmic formula and software. The dimensions were fractional, perfectly aligning with fractal dimensions.

Ultimately, this research found that the traditional Iranian artist intuitively understood fractal geometry (the geometry of nature) and abstractly expressed it in their works. This process was likely performed subconsciously by the artist, which can inspire architects to create modern and beautiful designs aligned with the human nature-oriented spirit. Since most patterns and nodes in decorations exhibit similar characteristics

Table 5. Comparison of fractal geometry and "dopanji tabldar" knot. Source: Authors.

Features of fractal geometry	Dopanji knot
Self-similarity: (repetition) (small scale)	 Fractal sample 
Dimension calculation with software.	 D=1.40  D=1.58  D=1.26

to those studied here, and because the nodes have fractional dimensions due to empty spaces and breaks, this research can serve as a comparative framework for other architectural decorations. However, for scientific caution, it is necessary to study and examine each case independently.

Endnotes

1-(born November 20, 1924 - died October 14, 2010) was a French-American mathematician who is known as the father of fractal geometry.
2- Aurangzeb Bahadur Alamgir, titled as Muhiuddin Muhammad, was the sixth Mughal Emperor of India, reigning from 1658 to 1707. Aurangzeb means 'adorned throne' in Persian, referring to the royal throne.

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